Physique mathématique : résumé

1 Magnetostatics

Biot and Savart law

$$d\vec{B} = k.\frac{I\vec{dl} \times \vec{x}}{|x|^3} \tag{1}$$

$$d\vec{F} = I\vec{dl} \times \vec{B} \tag{2}$$

$$\int_{\partial S} \vec{B}.\vec{dl} = \mu \int_S \vec{J}.\vec{dS} \tag{3}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}(\vec{x}) \tag{4}$$

Coulomb gauge

$$\Delta\vec{A} = -\mu_0\vec{J} \tag{5}$$

Magnetic moment

$$\vec{m} = \frac{1}{2}\int d^3x'(\vec{x}' \times \vec{J}(\vec{x}')) \tag{6}$$

Density of magnetic moment

$$\vec{\mathcal{M}}(x) = \frac{1}{2}(\vec{x} \times \vec{J}(x)) \tag{7}$$

$$\vec{B}(x) = \frac{\mu_0}{4\pi}\frac{3\vec{x}(\vec{m}\vec{x}) - \vec{m}.\vec{x}^2}{x^5} \tag{8}$$

Torque in classical mechanics

$$\vec{T} = \sum_i \vec{r}_i \times \vec{F}_i \tag{9}$$

Torque in magnetostatics

$$\vec{T} \simeq \vec{m} \times \vec{B} \tag{10}$$

Faraday induction law

$$\mathcal{E} = -\frac{d\phi}{dt} \tag{11}$$

with $\phi = \int_S \vec{B}.\vec{dS}$ and $\mathcal{E} = \int_{\partial S} \vec{E}.\vec{dl}$

Energy of magnetic field

$$\delta W = I.\delta\phi \tag{12}$$

$$W = \frac{1}{2\mu_0}\int d^3x.B^2 \tag{13}$$

$$W = \frac{1}{2}\int d^3x.\vec{J}.\vec{A} \tag{14}$$

2 Maxwell Equations

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0} \tag{I}$$

$$\vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J} \tag{II}$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \tag{III}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{IV}$$

Conservation law

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}.\vec{J} = 0 \tag{15}$$

Potentials

$$\vec{B} = \vec{\nabla}\times\vec{A} \tag{16}$$

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial\vec{A}}{\partial t} \tag{17}$$

Relativistic form

$$\partial_\mu F^{\mu\nu} = \frac{1}{c\epsilon_0}j^\nu \tag{18}$$

with $F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{pmatrix}$ and $j^\mu = (c\rho, \vec{J})$

3 Electric and magnetic fields in media

Polarization

$$\frac{P_{tot}}{V} = \sigma \equiv P \tag{19}$$

$$\rho_P(x) = -\vec{\nabla}.\vec{P}(x) \tag{20}$$

$$\sigma_P(x) = \vec{n}.\vec{P}(x) \tag{21}$$

$$\rho_{ext} = \vec{\nabla}.\vec{D} \tag{22}$$

with $\vec{D} = \epsilon_0\vec{E} + \vec{P}$

$$\vec{P} \simeq \epsilon_0\chi_e.\vec{E} \tag{23}$$

$$\vec{D} \simeq \epsilon\vec{E} \tag{24}$$

with $\epsilon = \epsilon_0(1 + \chi_e)$

$$\vec{\nabla}.\vec{E} \simeq \frac{\rho}{\epsilon} \tag{25}$$

Continuity

$$\vec{n}_{21} \times (\vec{E}_2 - \vec{E}_1) = 0 \tag{26}$$

$$\vec{n}_{21}.(\vec{D}_2 - \vec{D}_1) = \sigma_{ext} \tag{27}$$

Energy

$$W = \frac{1}{2}\int d^3x(\vec{E}.\vec{D}) \tag{28}$$

Magnetostatics in media

$$\vec{J}_M(x) = \vec{\nabla} \times \vec{M}(x) \tag{29}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_{ext} \tag{30}$$

with $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

Relation between B and H

$$\vec{B} = \mu\vec{H} \tag{31}$$

for isotropic diamagnetic and paramagnetic substances

$$\vec{B} = F(\vec{H}) \tag{32}$$

for ferromagnetic substances

4 Charged particle in electromagnetic field

Equation of motion

$$\frac{d\hat{p}^{\mu}}{ds} = \frac{e}{c^2}\hat{F}^{\mu\nu}.\hat{u}_{\nu} \tag{33}$$

– Space component :

$$\frac{d\vec{p}}{dt} = e[\vec{E} + \vec{v} \times \vec{B}] \tag{34}$$

with $\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

– Time component :

$$\frac{d\mathcal{E}}{dt} = e\vec{v}.\vec{E} \tag{35}$$

–

$$\mathcal{E} = \sqrt{m^2c^4 + p^2} \tag{36}$$

–

$$\vec{p} = \frac{\vec{v}\mathcal{E}}{c^2} \tag{37}$$

Gyration frequency

$$\vec{\omega}_B = \frac{e\vec{B}}{m\gamma} = \frac{ec^2\vec{B}}{\mathcal{E}} \tag{38}$$

Drift in non-uniform magnetic field

$$\vec{v}_{drift} = \frac{a^2}{2B_0}.\vec{\omega}_0 \times \vec{\nabla}B \tag{39}$$

Drift in a constant electric and magnetic field

$$\vec{v}_{drift} = \frac{\vec{E} \times \vec{B}}{B^2} \tag{40}$$

Field invariants

$$I_1 = c^2\vec{B}^2 - \vec{E}^2 \tag{41}$$

$$I_2 = \vec{E}.\vec{B} \tag{42}$$

5 Electromagnetic waves

D'Alembert equations

$$[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \partial_i^2]E_i = 0 \tag{43}$$

$$[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \partial_i^2]B_i = 0 \tag{44}$$

Monochromatic wave

$$\vec{E} = Re[\vec{\mathcal{E}}_0 e^{i\omega t - i\vec{k}.\vec{x}}] \tag{45}$$

$$\vec{B} = Re[\vec{\mathcal{B}}_0 e^{i\omega t - i\vec{k}.\vec{x}}] \tag{46}$$

with $\vec{k} = \frac{\vec{n}\omega}{c}$

Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \vec{n}.cW \tag{47}$$

Energy

$$W = \frac{1}{2}\frac{1}{\mu_0}\vec{B}^2 + \frac{1}{2}\epsilon_0\vec{E}^2 \tag{48}$$

$$-J.E = \frac{\partial w}{\partial t} + \vec{\nabla}.\vec{S} \tag{49}$$

Snell law

$$\frac{n}{n'} = \frac{sin\beta}{sin\alpha} \tag{50}$$

with $n = \frac{c}{v}$

Brewster angle

$$cos^2\alpha = \frac{1}{\frac{n'^2}{n^2} + 1} \tag{51}$$

6 Emission of electromagnetic waves

$$[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2]\varphi = \frac{\rho}{\epsilon_0} \tag{52}$$

$$[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2]\vec{A} = \mu_0\vec{J} \tag{53}$$

Potentiels retardés

$$\varphi(\vec{x}, t) = \frac{1}{4\pi\epsilon_0}\int dV \frac{\rho(t - \frac{|\vec{x'} - \vec{x}|}{c}, \vec{x})}{|\vec{x'} - \vec{x}|} \tag{54}$$

$$\vec{A}(\vec{x}, t) = \frac{\mu_0}{4\pi}\int dV \frac{\vec{J}(t - \frac{|\vec{x'} - \vec{x}|}{c}, \vec{x})}{|\vec{x'} - \vec{x}|} \tag{55}$$

Larmor formule

$$I = \frac{\mu_0 e^2}{6\pi c}\frac{\vec{a}^2 - (\frac{\vec{v}}{c} \times \vec{a})^2}{(1 - \frac{v^2}{c^3})^3} \tag{56}$$