

$$f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$$

Recherche de racine : $f(-1) = 0$.

$$\begin{aligned} f(x) &= (x+1) \cdot (ax^4 + bx^3 + cx^2 + dx + e) \\ &= ax^5 + ax^4 + bx^4 + bx^3 + cx^3 + cx^2 + dx^2 + dx + ex + e \\ &= ax^5 + (a+b) \cdot x^4 + (b+c) \cdot x^3 + (c+d) \cdot x^2 + (d+e) \cdot x + e \end{aligned}$$

On identifie :

$$\begin{cases} a=1 \\ a+b=1 \\ b+c=1 \\ c+d=1 \\ d+e=1 \\ e=1 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=0 \\ c=1 \\ d=0 \\ e=1 \end{cases}$$

$$\begin{aligned} f(x) &= (x+1) \cdot (x^4 + x^2 + 1) \\ &= (x+1) \cdot (x^4 + 2x^2 + 1 - x^2) \\ &= (x+1) \cdot ((x^2 + 1)^2 - x^2) \\ &= (x+1) \cdot (x^2 - x + 1)(x^2 + x + 1) \end{aligned}$$