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Noel Amenc, and Lionel Martellini

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Abstract

This paper attempts to evaluate the out-of-sample performance of an improved estimator of the covariance structure of hedge fund index returns, focusing on its use for optimal portfolio selection. Using data from CSFB-Tremont hedge fund indices, we find that ex-post volatility of minimum variance portfolios generated using implicit factor based estimation techniques is between 1.5 and 6 times lower than that of a value-weighted benchmark, such differences being both economically and statistically significant. This strongly indicates that optimal inclusion of hedge funds in an investor portfolio can potentially generate a dramatic decrease in the portfolio volatility on an out-of-sample basis. Differences in mean returns, on the other hand, are not statistically significant, suggesting that the improvement in terms of risk control does not necessarily come at the cost of lower expected returns.

*Noël Amenc is with the EDHEC Graduate School of Business and Misys Asset Management Systems. Lionel Martellini is with the Marshall School of Business at the University of Southern California. Research for this paper received the support of the EDHEC/MISYS multi-style/multi-class program. Please send all correspondence to Lionel Martellini, University of Southern California, Marshall School of Business, Business and Economics, Hoffman Hall 710, Los Angeles, CA 90089-1427. Phone: (213) 740 5796. Email address: martelli@usc.edu. We would like to express our gratitude to Vikas Agarwal, Hossein Kazemi, François-Serge Lhabitant, Terry Marsh, George Martin, and Narayan Naik for very useful comments and suggestions. We also thank Jérôme Maman and Mathieu Vaissié for superb computer assistance. All errors are, of course, the authors' sole responsibility.

1 Introduction

A dramatic change has occurred in recent years in the attitude of institutional investors, banks and the traditional fund houses towards alternative investment in general, and hedge funds in particular. Interest is undoubtedly gathering pace, and the consequences of this potentially significant shift in investment behavior are far-reaching, as can be seen from the conclusion of a recent research survey about the future role of hedge funds in institutional asset management (Gollin/Harris Ludgate survey (2001)): "Last year it was evident (...) that hedge funds were on the brink of moving into the mainstream. A year on, it is safe to argue that they have arrived". According to this survey, 64% of European institutions for which data was collected currently invest, or were intending to invest, in hedge funds (this figure is up from 56% in 2000). Interest is also growing in Asia, and of course in the United States, where the hedge fund industry was originated by Alfred Jones back in 1949. As a result, the value of the hedge fund industry is now estimated at more than 500 billion US dollars, with more than 5,000 funds worldwide (Frank Russell - Goldman Sachs survey (1999)), and new hedge funds are being launched every day to meet the surging demand.

Among the reasons that explain the growing institutional interest in hedge funds, there is first an immediate and perhaps superficial one: hedge funds always gain in popularity when equity market bull runs end, as long-only investors seek protection on the downside. This certainly explains in part the rising demand for hedge funds in late 2000 and early 2001. A more profound reason behind the growing acceptance of hedge funds is the recognition that they can offer a more sophisticated approach to investing through the use of derivatives and shortselling, which results in low correlations with traditional asset classes. Furthermore, while it has been documented that international diversification fails when it is most needed, i.e., in periods of crisis (see for example Longin and Solnik (1995)), there is some evidence that conditional correlations of at least some hedge strategies with respect to stock and bond market indexes tend to be stable across various market conditions (Schneeweis and Spurgin (1999)).[1]

A classic way to analyze and formalize the benefits of investing in hedge funds is to note the improvement in the risk-return trade-off they allow when included in a traditional long-only stock and bond portfolio. Since seminal work by Markowitz (1952), it is well-known that this trade-off can be expressed in terms of mean-variance analysis under suitable assumptions on

[1] In a follow up paper, Schneeweis and Spurgin (2000) find that different strategies exhibit different patterns. They make a distinction between good, bad and stable correlation depending whether correlation is higher (resp. lower, stable) in periods of market up moves compared to periods of market down moves. Agarwal and Narayan (2001) also report evidence of higher correlation between some hedge fund returns and equity market returns when conditioning upon equity market down moves as opposed to conditioning upon up moves.

investor preferences (quadratic preferences) or asset return distribution (normal returns).[2] In the academic and practitioner literature on the benefits of alternative investment strategies, examples of enhancement of long-only efficient frontiers through optimal investments in hedge fund portfolios abound (see for example Schneeweis and Spurgin (1999) or Karavas (2000)).

One problem is that, to the best of our knowledge, all these papers only focus on *in-sample* diversification results of standard sample estimates of covariance matrix. In sharp contrast with the large amount of literature on asset return covariance matrix estimation in the traditional investment area, there has actually been very little scientific evidence evaluating the performance of different portfolio optimization methods in the context of alternative investment strategies. This is perhaps surprising given that the benefits promised by portfolio optimization critically depend on how accurately the first and second moments of hedge fund return distribution can be estimated. This paper attempts to fill in this gap by evaluating the out-of-sample performance of an improved estimator of the covariance structure of hedge fund index returns, focusing on its use for optimal portfolio selection.

It has since long been recognized that the sample covariance matrix of historical returns is likely to generate high sampling error in the presence of many assets, and several methods have been introduced to improve asset return covariance matrix estimation. One solution is to impose some structure on the covariance matrix to reduce the number of parameters to be estimated. Several models fall within that category, including the constant correlation approach (Elton and Gruber (1973)), the single factor forecast (Sharpe (1963)) and the multi-factor forecast (e.g., Chan, Karceski and Lakonishok (1999)). In these approaches, sampling error is reduced at the cost of some specification error. Several authors have studied the optimal trade-off between sampling risk and model risk in the context of optimal shrinkage theory. This includes optimal shrinkage towards the grand mean (Jorion (1985, 1986)), optimal shrinkage towards the single-factor model (Ledoit (1999)). Also related is a recent paper by Jagannathan and Ma (2000) who show that imposing weight constraints is actually equivalent to shrinking the extreme covariance estimates to the average estimates. In this paper, we consider an implicit factor model in an attempt to mitigate model risk and impose *endogenous* structure. The advantage of that option is that it involves low specification error (because of the "let the data talk" type of approach) and low sampling error (because some structure

[2] There is clear evidence that hedge fund returns may not be normally distributed (see for example Amin and Kat (2001) or Lo (2001)). Hedge funds typically exhibit non-linear option-like exposures to standard asset classes (Fung and Hsieh (1997a, 2000), Agarwal and Naik (2000)) because they can use derivatives, follow all kinds of dynamic trading strategies, and also because of the explicit sharing of the upside profits (post-fee returns have option-like element even if pre-fee returns do not). As a result, hedge fund returns may not be normally distributed even if traditional asset returns were. Fung and Hsieh (1997b) argue, however, that mean-variance analysis may still be applicable to hedge funds as a second-order approximation as it essentially preserves the ranking of preferences in standard utility functions.

is imposed). Implicit multi-factor forecasts of asset return covariance matrix can be further improved by noise dressing techniques and optimal selection of the relevant number of factors (see section 2).

We choose to focus on the issue of estimating the covariances of hedge fund returns, rather than expected returns, for a variety of reasons. First, there is a general consensus that expected returns are difficult to obtain with a reasonable estimation error. What makes the problem worse is that optimization techniques are very sensitive to differences in expected returns, so that portfolio optimizers typically allocate the largest fraction of capital to the asset class for which estimation error in the expected returns is the largest. On the other hand, there is a common impression that return variances and covariances are much easier to estimate from historical data. Since early work by Merton (1980) or Jorion (1985, 1986), it has been argued that the optimal estimator of the expected return is noisy with a finite sample size, while the estimator of the variance converges to the true value as the data sampling frequency is increased. As a result, we approach the question of optimal strategic asset allocation in the alternative investment universe in a pragmatic manner. Because of the presence of large estimation risk in the estimated expected returns, we evaluate the performance of an improved estimator for the covariance structure of hedge fund returns, focusing on its use for selecting the one portfolio on the efficient frontier for which no information on expected returns is required, the minimum variance portfolio.[3]

In particular, we consider a portfolio invested only in hedge funds and an equity-oriented portfolio invested in traditional equity indices and equity-related alternative indices. Our methodology for testing minimum variance portfolios is similar to the one used in Chan et al. (1999) and Jagannathan and Ma (2000): we estimate sample covariances over one period and then generate out-of-sample estimates. Using data from CSFB-Tremont hedge fund indices, we find that ex-post volatility of minimum variance portfolios generated using implicit factor based estimation techniques is between 1.5 and 6 times lower than that of a a value-weighted benchmark (the S&P 500), such differences being both economically and statistically significant. This strongly indicates that optimal inclusion of hedge funds in an investor portfolio can potentially generate a dramatic decrease in the portfolio volatility on an out-of-sample basis. Differences in mean returns, on the other hand, are not statistically significant, suggesting that the improvement in terms of risk control does not necessarily come at the cost of lower expected returns.

The rest of the paper is organized as follows. In Section 2, we introduce the implicit factor approach to asset return covariance estimation. In Section 3, we present the data, the methodology and the results. Section 4 concludes.

[3] Alternatively, one motivation in focusing on the minimum variance portfolio is to note that it is the efficient portfolio obtained under the null hypothesis of no informative content in the cross-section of expected returns.

2 Covariance Matrix Estimation

Several solutions to the problem of asset return covariance matrix estimation have been suggested in the traditional investment literature. The most common estimator of return covariance matrix is the sample covariance matrix of historical returns

$$S = \frac{1}{T-1}\sum_{t=1}^{T}\left(h_t - \overline{h}\right)\left(h_t - \overline{h}\right)'$$

where T is the sample size, h_t is a $N \times 1$ vector of hedge fund returns in period t, N is the number of assets in the portfolio, and $\overline{h}$ is the average of these return vectors. We denote by S_{ij} the (i, j) entry of S.

A problem with this estimator is typically that a covariance matrix may have too many parameters compared to the available data. If the number of assets in the portfolio is N, there are indeed $\frac{N(N-1)}{2}$ different covariance terms to be estimated. The problem is particularly acute in the context of alternative investment strategies, even when a limited set of funds or indexes are considered, because data is scarce given that hedge fund returns are only available on a monthly basis.

One possible cure to the curse of dimensionality in covariance matrix estimation is to impose some structure on the covariance matrix to reduce the number of parameters to be estimated. In the case of asset returns, a low-dimensional linear factor structure seems natural and consistent with standard asset pricing theory, as linear multi-factor models can be economically justified through equilibrium arguments (cf. Merton's Intertemporal Capital Asset Pricing Model (1973)) or arbitrage arguments (cf. Ross's Arbitrage Pricing Theory (1976)). Therefore, in what follows, we shall focus on K-factor models with uncorrelated residuals.[4] Of course, this leaves two very important questions: *how much structure should we impose?* (the fewer the factors, the stronger the structure) and *what factors should we use?* A standard trade-off exists between model risk and estimation risk. The following options are available:

- Impose no structure. This choice involves low specification error and high sampling error, and led to the use of the sample covariance matrix.[5]

- Impose some structure. This choice involves high specification error and low sampling error. Several models fall within that category, including the constant correlation approach (Elton and Gruber (1973)), the single factor forecast (Sharpe (1963)) and the multi-factor forecast (e.g., Chan, Karceski and Lakonishok (1999)).

[4] Another way to impose structure on the covariance matrix is the constant correlation model (Elton and Gruber (1973)). This model can actually be alternatively thought of as a James-Stein estimator that shrinks each pairwise correlation to the global mean correlation.

[5] One possible generalization/improvement to this sample covariance matrix estimation is to allow for declining weights assigned to observations as they go further back in time (Litterman and Winkelmann (1998)).

- Impose optimal structure. This choice involves medium specification error and medium sampling error. The optimal trade-off between specification error and sampling error has led either to an optimal shrinkage towards the grand mean (Jorion (1985, 1986)) or an optimal shrinkage towards the single-factor model (Ledoit (1999)), or to the introduction of portfolio constraints (Jagannathan and Ma (2000)).

In this paper, we consider an implicit factor model in an attempt to mitigate model risk and impose *endogenous* structure. The advantage of that option is that it involves low specification error (because of the "let the data talk" type of approach) and low sampling error (because some structure is imposed). Implicit multi-factor forecasts of asset return covariance matrix can be further improved by noise dressing techniques and optimal selection of the relevant number of factors (see below).

More specifically, we use Principle Component Analysis (PCA) to extract a set of implicit factors. The PCA of a time-series involves studying the correlation matrix of successive shocks. Its purpose is to explain the behavior of observed variables using a smaller set of unobserved implied variables. Since principal components are chosen solely for their ability to explain risk, a given number of implicit factors always capture a larger part of asset return variance-covariance than the same number of explicit factors. One drawback is that implicit factors do not have a direct economic interpretation (except for the first factor, which is typically highly correlated with the market index). Principal component analysis has been used in the empirical asset pricing literature (see for example Litterman and Scheinkman (1991), Connor and Korajczyk (1993) or Fedrigo, Marsh and Pfleiderer (1996), among many others).

From a mathematical standpoint, it involves transforming a set of N correlated variables into a set of orthogonal variables, or implicit factors, which reproduces the original information present in the correlation structure. Each implicit factor is defined as a linear combination of original variables. Define H as the following matrix

$$H = (h_{it})_{\substack{1\leq t\leq T \\ 1\leq i\leq N}}$$

We have N variables h_i $i = 1, ..., N$, i.e., monthly returns for N different hedge fund indexes, and T observations of these variables.[6] PCA enables us to decompose h_{tk} as follows[7]

$$h_{tk} = \sum_{i=1}^{N} \sqrt{\lambda_i} U_{ik} V_{ti}$$

where

$(U) = (U_{ik})_{1\leq i,k\leq N}$ is the matrix of the N eigenvectors of $H'H$.

[6] The asset returns have first been normalized to have zero mean and unit variance.

[7] For an explanation of this decomposition in a financial context, see for example Barber and Copper (1996).

$(U^\intercal) = (U_{ki})_{1\leq k,i\leq N}$ is the transposed of U.

$(V) = (V_{ti})_{\substack{1\leq t\leq T \\ 1\leq i\leq N}}$ is the matrix of the N eigenvectors of HH'.

Note that these N eigenvectors are orthonormal. λ_i is the eigenvalue (ordered by degree of magnitude) corresponding to the eigenvector U_i. Denoting $s_{ik} = \sqrt{\lambda_i} U_{ik}$ the principal component sensitivity of the k^{th} variable to the i^{th} factor, and $V_{ti} = F_{ti}$, one can equivalently write

$$h_{tk} = \sum_{i=1}^{N} s_{ik} F_{ti}$$

where the N factors F_i are a set of orthogonal variables. The main challenge is to describe each variable as a linear function of a reduced number of factors. To that end, one needs to select a number of factors K such that the first K factors capture a large fraction of asset return variance, while the remaining part can be regarded as statistical noise

$$h_{tk} = \sum_{i=1}^{K} \sqrt{\lambda_i} U_{ik} V_{ti} + \varepsilon_{tk} = \sum_{i=1}^{K} s_{ik} F_{ti} + \varepsilon_{tk}$$

where some structure is imposed by assuming that the residuals ε_{tk} are uncorrelated one to another. The percentage of variance explained by the first K factors is given by $\frac{\sum_{i=1}^{K} \lambda_i}{\sum_{i=1}^{N} \lambda_i}$.

A sophisticated test by Connor and Corajczyk (1993) finds between 4 and 7 factors for the NYSE and AMEX over 1967-1991, which is roughly consistent with Roll and Ross (1980). Ledoit (1999) uses a 5 factor model. In this paper, we select the relevant number of factors by applying some explicit results from the theory of random matrices (see Marchenko and Pastur (1967)).[8] The idea is to compare the properties of an empirical covariance matrix (or equivalently correlation matrix since asset returns have been normalized to have zero mean and unit variance) to a null hypothesis purely random matrix as one could obtain from a finite time-series of strictly independent assets. It has been shown (see Johnstone (2001) for a recent reference and Laloux et al. (1999) for an application to finance) that the asymptotic density of eigenvalues λ of the correlation matrix of strictly independent asset reads

$$f(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{\max} - \lambda)(\lambda_{\min} - \lambda)}}{\lambda} \tag{1}$$

where $Q = T/N$ and

$$\begin{aligned} \lambda_{\max} &= 1 + \frac{1}{Q} + 2\sqrt{\frac{1}{Q}} \\ \lambda_{\min} &= 1 + \frac{1}{Q} - 2\sqrt{\frac{1}{Q}} \end{aligned}$$

Theoretically speaking, this result can be exploited to provide formal testing of the assumption that a given factor represents information and not noise. However, the result is an

[8] Another decision rule would be: keep sufficient factors to explain x% of the covariation in the portfolio.

asymptotic result that can not be taken at face value for a finite sample size. One of the most important features predicted by equation (1) is the fact that the lower bound of the spectrum $\lambda_{\min}$ is strictly positive (except for $Q = 1$), and therefore, there are no eigenvalues between 0 and $\lambda_{\min}$. We use a conservative interpretation of this result to design a systematic decision rule and decide to regard as statistical noise all factors associated with an eigenvalue lower than $\lambda_{\max}$. In other words, we take $K = \{i \text{ such that } \lambda_i > \lambda_{\max} \text{ and } \lambda_{i+1} < \lambda_{\max}\}$.[9]

3 Empirical Results

The hedge fund universe is made up of more than 5,000 funds. We focus on hedge fund index returns rather than hedge fund returns because the lack of liquidity and standard lock-up periods typical to hedge fund investing makes the computation of efficient frontiers for individual hedge funds from historical returns not particularly relevant from a forward-looking investment perspective. On the other hand, there are more and more investible benchmarks designed to track the performance of hedge fund indexes (see Amenc and Martellini (2001)). As a result, generating efficient frontiers on the basis of hedge fund indexes and sub-indexes is more than a simple academic exercise. We consider both a portfolio of hedge fund indexes, and a portfolio mixing traditional and alternative investment vehicles.

3.1 Data and Methodology

There are at least a dozen of competing hedge fund index providers, and they provide a somewhat contrasted picture of hedge fund returns (see Amenc and Martellini (2001) or Fung and Hsieh (2001)). To represent the alternative investment universe, we choose in this paper to use data from Credit Swiss First Boston - Tremont (CSFB-Tremont). The CSFB/Tremont Hedge Fund indexes have been used in a variety of studies on hedge fund performance (e.g., Lhabitant (2001)) and offer several advantages with respect to their competitors:

- They are transparent both in their calculation and composition, and constructed in a disciplined and objective manner. Starting from the TASS+ database, which tracks over 2,600 US and offshore hedge funds, the indexes only retain hedge funds that have at least US $10 million under management and provide audited financial statements. Only about 300 funds pass the screening process. The indexes are calculated on a monthly basis, and funds are re-selected on a quarterly basis as necessary. Funds are not removed from the indexes until they are liquidated or fail to meet the financial reporting requirements.

[9] In case no factor is such that the associated eigenvalue is greater than lambda max, we take $K = 1$, i.e., we retain the first component as the only factor.

- They are computed on a monthly basis and are currently the industry's only asset-weighted hedge fund indexes.[10] Funds are reselected quarterly to be included in the index, and in order to minimize the survivorship bias, they are not excluded until they liquidate or fail to meet the financial reporting requirements. This makes these indexes representative of the various hedge funds investment styles and useful for tracking and comparing hedge fund performance against other major asset classes.

The CSFB/Tremont sub-indexes have been launched in 1999 with data going back to 1994. They cover nine strategies: convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, fixed-income arbitrage, global macro, long/short equity and managed futures (see the Appendix for descriptive information on these nine strategies). To ensure that the results we obtain are sufficiently robust, we have also tested a couple of other hedge fund index providers (HFR and Zurich, respectively) and find very similar results that we do not report here.[11]

Table 1 reports correlations, means and standard deviations for the nine Tremont hedge fund sub-indexes based on monthly data over the period 1994-2000. We expect the benefits of diversification within the hedge fund universe to be significant because of the presence of low, and even negative, correlations between various hedge fund sub-indexes. We also confirm that the hedge fund universe is very heterogeneous: some hedge fund strategies have relatively high volatility (e.g. dedicated short bias, emerging markets, global macro, long/short equity and managed futures); they act as return enhancers and can be used as a substitute for some fraction of the equity holdings in an investor's portfolio. On the other hand, other hedge fund strategies have lower volatility (e.g., convertible arbitrage, equity market neutral, fixed-income arbitrage and event driven); they can be regarded as a substitute for some fraction of the fixed-income or cash holdings in an investor's portfolio.[12]

In the traditional investment universe, we choose well-known equity indexes, S&P growth, S&P value, S&P mid-cap and S&P small cap.

More specifically, we consider the following two investment universes.

- An alternative portfolio invested in the nine Tremont sub-indexes, i.e., convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, fixed-income arbitrage, global macro, long-short equity and managed futures.

[10] It should be noted that, as a result of the capitalization weighting and the bull market of the late nineties, the CSFB-Tremont indexes tend to be overweighted towards equity and equity exposure.

[11] These results can be obtained from the authors upon request.

[12] See Cvitanic et al. (2001b) for a formalization of the intuition that high (respectively, low) beta hedge funds can be regarded as natural substitutes for a fraction of an investor's equity (respectively, risk-free asset) portfolio holdings.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Convertible Arbitrage (1)	1.00								
Dedicated Short Bias (2)	-0.26	1.00							
Emerging Markets (3)	0.38	-0.56	1.00						
Equity Market Neutral (4)	0.35	-0.43	0.25	1.00					
Event Driven (5)	0.60	-0.65	0.70	0.43	1.00				
Fixed-Income Arbitrage (6)	0.65	-0.08	0.33	0.06	0.44	1.00			
Global Macro (7)	0.31	-0.14	0.42	0.23	0.40	0.47	1.00		
Long-Short Equity (8)	0.28	-0.77	0.60	0.37	0.67	0.23	0.45	1.00	
Managed Futures (9)	-0.35	0.23	-0.11	0.19	-0.23	-0.22	0.26	-0.05	1.00
Mean	0.85	0.08	0.54	0.94	0.96	0.54	1.14	1.31	0.49
Standard Deviation	1.46	5.52	5.92	0.99	1.91	1.26	4.14	3.65	3.31

Table 1: Descriptive statistics for the Tremont hedge fund indexes. This table reports correlations, means and standard deviations for the nine Tremont hedge fund indexes and the Tremont global index based on monthly data over the period 1994-2000.

- An equity-oriented portfolio invested in S&P growth, S&P value, S&P mid-cap and S&P small cap for the traditional part, and in three equity-oriented Tremont indexes, dedicated market bias, long short equity and equity market neutral, for the alternative part.

Because of the presence of large estimation risk in the estimated expected returns, we choose to evaluate the performance of the improved estimator for the covariance structure of hedge fund returns by focusing on its use for selecting the minimum variance portfolio, the only portfolio on the efficient frontier for which no estimation of expected returns is needed. Our methodology for testing minimum variance portfolios is similar to the one used in Chan et al. (1999) and Jagannathan and Ma (2000).

3.2 Alternative Investment Universe

We use the previous 48 months of observations (beginning of 1994 to end of 1998) to estimate the covariance matrix of the returns of the 9 hedge fund sub-indexes. We then form two versions of global minimum variance portfolios: the nonnegativity constrained and the one with both nonnegativity constraint and a tracking error constraint. These portfolios are held for 6 months, their monthly returns are recorded, and the same process is repeated again. So, minimum variance portfolios have ex-post monthly returns from early 1999 to the end of 2000. The means and variances of these portfolios are used to assess the performance of optimal diversification.

Table 2 reports ex-post means, standard deviations, and other characteristics of the global minimum variance portfolio. In addition to the global minimum variance portfolio, we also considered the following two portfolios: the value-weighted Tremont global index and the equally-weighted portfolio of the various indices.

	Mean Return	Std Deviation	Skewness	Kurtosis	Max. Weight
Minimum variance portfolio	12.16	1.57	-0.03	1.91	0.49
Equally weighted index	9.13	4.79	0.43	8.97	0.14
Global Tremont index	12.50	9.95	0.59	3.03	N/A

Table 2: Ex-post mean, standard deviation and other characteristics of the minimum variance portfolio. Mean and standard deviation are expressed in percentage per year, and obtained from monthly data though a multiplicative factor of 12 and square-root of twelve, respectively.

We find that ex-post volatility of minimum variance portfoliois almost 3 times lower than that of a naively diversified equally-weighted portfolio, and almost 7 times lower than that of the value-weighted Global Tremont Index. Interestingly, the mean return on the minimum variance portfolio tends to dominate that of the equally-weighted. The annual mean return on the minimum variance portfolio is 12.16%, which compares to 9.13% and 12.50% for the equally-weighted and value-weighted portfolios, respectively. Such differences in mean returns may not, however, be statistically significant. To test whether there is a statistically significant difference in the ex-post returns and ex-post return variances, we test the equalities of mean returns and mean squared returns. These results can be found in table 3.

	T-Test Mean	T-Test Mean Squared
Equally weighted index	0.11	-1.39
Global Tremont index	-0.16	**-2.39**

Table 3: T-tests of equal means and mean squared returns for the minimum variance portfolio. This table reports t-tests of equal mean and mean squared returns of the minimum variance portfolio. We test whether its mean and mean squared returns are statistically different from those obtained from benchmark portfolios. Values significant at the 5 percent level appear boldfaced.

As we suspected, differences in mean returns are not significant. On the other hand, from the numbers in table 3, we conclude that the Global Tremont Index has significantly higher mean squared returns than the minimum variance portfolio. Because of concern over non normality of hedge fund returns, we also compute skewness and kurtosis of the portfolio returns respectively defined as

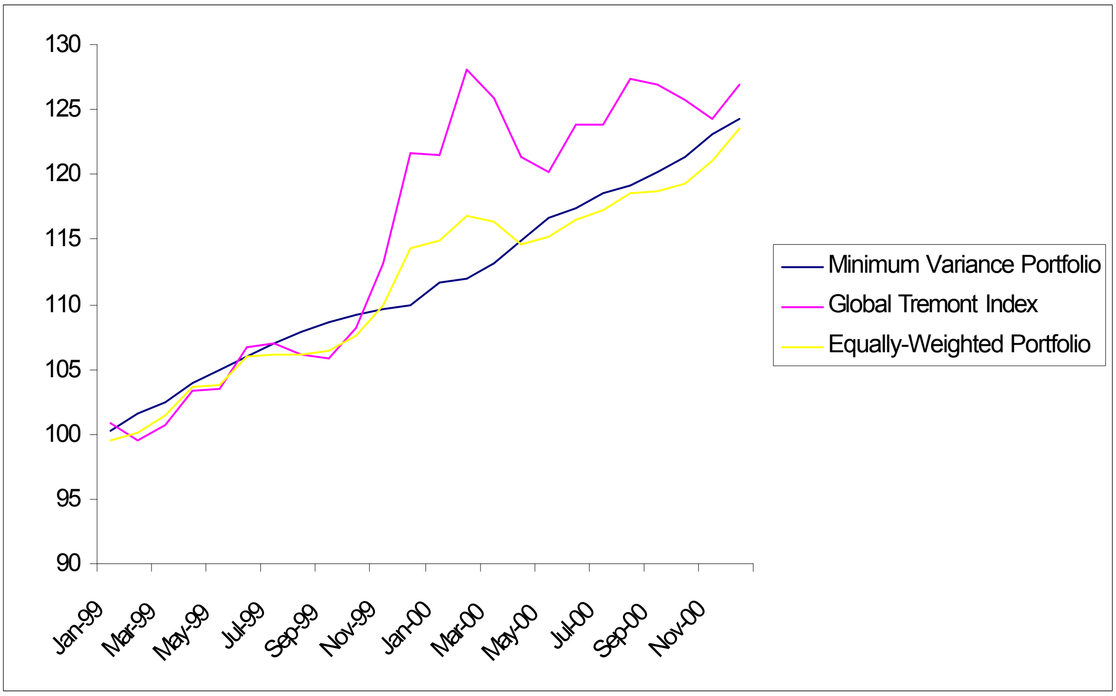

Figure 1: This graph displays the evolution of \$100 invested in January 1999 in the Global Tremont Index, an equally-weighted portfolio of Tremont indexes and the minimum variance portfolio obtained from an implicit factor-based variance-covariance matrix estimator, where all factors with eigenvalues lower than $\lambda_{\max}$ are treated as noise.

$$sk = \frac{\text{sample mean of } (x_i - \mu_i)^3}{\sigma_i^3}$$
$$ku = \frac{\text{sample mean of } (x_i - \mu_i)^4}{\sigma_i^4}$$

where μ_i is the sample mean of x_i, σ_i is the standard deviation of x_i and we check that the decrease in volatility is not matched by a significant shift in either skewness or kurtosis (see table 2).

As an illustration, figure 1 displays the evolution of \$100 invested in January 1999 in the Global Tremont Index, an equally-weighted portfolio of Tremont indexes and the minimum variance portfolio obtained from an implicit factor-based variance-covariance matrix estimator, where all factors with eigenvalues lower than $\lambda_{\max}$ are treated as noise. As can be seen from the figure, the minimum variance portfolio has a much smoother path than its equally-weighted and value-weighted counterparts.

By focusing on minimizing the variance, we might expect that the portfolios tend to over-emphasize the low volatility relative value and arbitrage strategies. Table 4, which contains the dynamics of portfolio weights, allows us to check that this has happened to a certain extend.

	01/01/1999	07/01/1999	01/01/2000	07/01/2000
Convertible Arbitrage	0.218	0.227	0.227	0.168
Dedicated Short Bias	0.102	0.099	0.092	0.085
Emerging Markets	0.000	0.000	0.000	0.000
Equity Market Neutral	0.398	0.404	0.459	0.486
Event Driven	0.014	0.002	0.000	0.023
Fixed Income Arbitrage	0.206	0.214	0.179	0.188
Global Macro	0.000	0.000	0.000	0.000
Long/Short	0.000	0.000	0.000	0.000
Managed Futures	0.062	0.054	0.044	0.050

Table 4: Portfolio weights. This table reports weights of the minimum variance portfolios obtained from an implicit factor-based variance-covariance matrix estimator.

First, we note that there are 3 strategies, emerging markets, global macro and long/short which are never included in the minimum variance portfolio. Not surprisingly, these are the strategies, alongside with dedicated short bias which also does not get much weighting, associated with the most volatile returns (see table 1). Conversely, we find that the largest fraction of the portfolio is consistently invested in equity market neutral, which has a low 0.99% monthly volatility over the period 1994-2000.[13] These results are also consistent with previous research (Schneeweis and Spurgin (2000)) that has suggested that hedge fund strategies fall into the following two types: return enhancer and risk diversifier. Using in-sample efficient frontiers, Schneeweis and Spurgin (2000) find that a typical investor holding a stock/bond portfolio should expect return enhancement more than risk diversification when investing in emerging markets, global macro or long/short strategies. This is confirmed by the out-of-sample results of the minimum variance optimization.

3.3 Global Investment Universe

The benefits of alternative investments can be better understood when hedge funds are combined with traditional assets in a diversified portfolio. To test whether optimal diversification can be achieved, we perform the following experiment. We consider an equity-oriented benchmark, invested in S&P 500 growth, S&P 500 value, S&P 400 mid-cap and S&P 600 small cap for the traditional part, and in Tremont dedicated short bias, Tremont market neutral and

[13] To get closer to the typical allocation to hedge funds, which emphasizes equity based strategies, an investor may want to include specific constraints on weights.

Tremont long/short for the alternative part.[14] From table 5, which displays an overview of pairwise correlations for these 7 indexes, we find, as expected, that the Tremont dedicated short bias index has a strong negative correlation with traditional equity indexes. This suggests that significant diversification benefit might be generated from the inclusion of that asset class in an equity portfolio.[15]

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
S&P 500 Growth (1)	1.00						
S&P 500 Value (2)	0.73	1.00					
S&P Mid Cap 400 (3)	0.73	0.82	1.00				
S&P Small Cap 600 (4)	0.61	0.62	0.87	1.00			
Tremont dedicated short bias (5)	-0.79	-0.64	-0.84	-0.84	1.00		
Tremont market neutral and (6)	0.43	0.48	0.47	0.41	-0.43	1.00	
Tremont long/short (7)	0.66	0.49	0.74	0.81	-0.77	0.37	1.00

Table 5: Descriptive statistics for the equity-oriented benchmark. This table reports pairwise correlations for the traditional and alternative equity-oriented indexes based on monthly data over the period 1994-2001.

To test the robustness of the approach, we have changed (decreased) the calibration period to 3 years (early 1994 to end of 1996), and extended the backtesting period to December 2001.[16] We compare the minimum variance portfolio to the following two portfolios: the S&P 500 on the one hand, and an equally-weighted portfolio invested in S&P 500 growth, S&P 500 value, S&P 400 mid-cap and S&P 600 small-cap for the traditional part, and in Tremont dedicated short bias, Tremont market neutral and Tremont long/short for the alternative part.

There are two possible approaches to the optimal allocation problem of an investor in the presence of both traditional and alternative investment styles. One approach focuses on minimizing *absolute* risk as measured by volatility. Another approach is *relative* risk control, where a constraint is imposed so as to prevent the investor portfolio to deviate too significantly

[14] While those strategies do significantly invest in equities, we do not include emerging markets, convertible arbitrage and event driven funds in the equity-oriented universe, our motivation being to keep the number of alternative styles comparable to the number of traditional styles. Several robustness checks that we have performed but do not report here clearly suggest that the choice of which indices to include in the portfolio does not qualitatively affect the results.

[15] In general, net short-biased directional equity hedge fund investing is not really a viable strategy. This is because equity short-biased funds that are negatively correlated with traditional equity indexes have significant negative carry. It is possible to have hedge fund strategies that are net negatively correlated (being more negatively correlated in down markets than in up markets) with positive carry, such as if the fund is long realized volatility. We are indebted to George Martin for these comments.

[16] We have also tested 24 months calibration periods, and obtain similar results.

from a given benchmark. This is consistent with common practice in the industry where the performance of an active portfolio is typically benchmarked against that of a broad-based market index.[17] In this section, we implement both the absolute and relative minimum variance optimization approaches, where we impose, respectively, a 5%, 10% and 15% tracking error constraints with respect to the S&P500.

Table 6 reports the ex-post mean, standard deviation and other characteristics of these constrained minimum variance portfolios, as well as similar performance measures for the S&P 500 and a naively diversified equally-weighted portfolio.

	Mean Return (TE=5%, 10%, 15%, ∞)	Volat. (TE=5%, 10%, 15%, ∞)
Min Var portfolio	10.03%, 8.74%, 10.39%, 11.55%	11.65%, 6.16%, 3.01%, 2.37%
Equally weighted index	11.63%	9.60%
S&P 500	11.80%	17.90%

Table 6: Ex-post mean, standard deviation and other characteristics of the contrained minimum variance portfolios and benchmark portfolios. TE= ∞ denotes the case of no tracking error constraint. Mean and standard deviation are expressed in percentage per year, and obtained from monthly data though a multiplicative factor of 12 and square-root of twelve, respectively.

Again, we find that optimal variance minimization allows an investor to achieve significantly lower portfolio variance. In particular, the annual volatility of the minimum variance portfolio is almost 3 times lower than that of the S&P 500 for a 10% tracking error and 6 times lower for a 15% tracking error. When no tracking error constraint is imposed, the ex-post volatility of the minimum variance portfolio is 2.37%, as opposed to 17.90% for the S&P 500. (In this table, the case of no tracking error constraint is denoted by $TE = \infty$.) These differences are statistically significant, as can be seen from t-statistics reported in table.7 On the other hand, we find, again, that differences in mean returns are not statistically significant.

These results suggest that the presence of tracking error constraints allows an investor to take advantage of optimal inclusion of hedge funds in an equity portfolio to potentially generate a dramatic decrease in the portfolio volatility on an out-of-sample basis while maintaining a reasonable exposure to traditional investment styles.

In the case of a stringent tracking error constraint (5% target), the volatility is still significantly lower than that of the S&P 500, while being higher than that of an equally-weighted

[17] Imposing a tracking error constraint with respect to a broad based index is actually very similar in spirit to mixing a global minimum variance portfolio with the market index, a practice that has sound theoretical foundations. From the two funds separation theorem, we know that a combination of the minimum variance portfolio and the market portfolio, proxied by the broad-based index, is also an efficient portfolio (see for example Ingersoll (1987)).

T-Statistics	Mean: TE=5%, 10%, 15%, ∞	Mean Squared: TE=5%, 10%, 15%, ∞
Equ.-weighted index	-0.69, 1.03, -0.33, -0.02	2.77, **-3.63**, **-5.17, -5.11**
S&P 500	-0.6, 0.52, -0.19, -0.03	**-5.50**, **-5.84**, **-6.06, -6.04**

Table 7: T-tests of equal mean and mean squared returns for the minimum variance portfolio. This table reports t-statistics for tests of equal mean and mean squared returns of the minimum variance portfolios compared to what is obtained from benchmark portfolios. TE= ∞ denotes the case of no tracking error constraint. Values significant at the 5 percent level appear boldfaced.

portfolio. From table 8, which contains information over the dynamics of portfolio weights in the 5% tracking error case, we also observe that two alternative investment styles play an important role in the diversification process. The optimal portfolio actually contains a significant fraction of the equity market neutral index, which posts a low volatility over the period, and also a fair share invested in the dedicated short biased index. This suggests that both the variance and covariance structure of hedge fund return with respect to traditional asset classes allows for an improvement in the volatility of a diversified portfolio.

	01/97	07/97	01/98	07/98	01/99	07/99	01/00	07/00	01/01	07/0
S&P500 Growth	30.38	28.14	29.68	31.07	37.47	38.58	39.35	41.31	38.98	40.0
S&P500 Value	23.03	25.82	33.20	35.14	36.54	35.07	35.42	32.88	32.47	33.4
S&P400 Mid Cap	12.79	12.91	5.97	1.16	2.35	0.00	0.39	2.50	0.79	1.3
S&P600 Small Cap	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.0
Ded. Short Bias	17.68	17.71	15.26	11.21	8.24	6.48	7.33	6.90	2.53	3.2
Eq. Market Neutral	14.44	15.30	15.89	21.41	15.39	17.17	16.06	16.42	25.22	21.9
Long/Short	1.68	0.13	0.00	0.00	0.00	2.70	1.45	0.00	0.00	0.0

Table 8: Portfolio weights. This table reports weights of the 5% TE constrained minimum variance portfolios obtained from an implicit factor-based variance-covariance matrix estimator.

It should also be noted that a very small fraction of the portfolio is invested in S&P Mid Cap and S&P Small Cap styles, which is largely due to the specific choice of the benchmark (S&P 500).

As an illustration, figure 2 displays the evolution of $100 invested in January 1997 in the S&P 500, an equally-weighted portfolio invested in S&P 500 growth, S&P 500 value, S&P 400 mid-cap and S&P 600 small cap for the traditional part, and in Tremont dedicated short bias, Tremont market neutral and Tremont long/short, and the minimum variance portfolios obtained from an implicit factor-based variance-covariance matrix estimator with a 5%, 10% and 15% tracking error constraints, and without tracking error constraint.

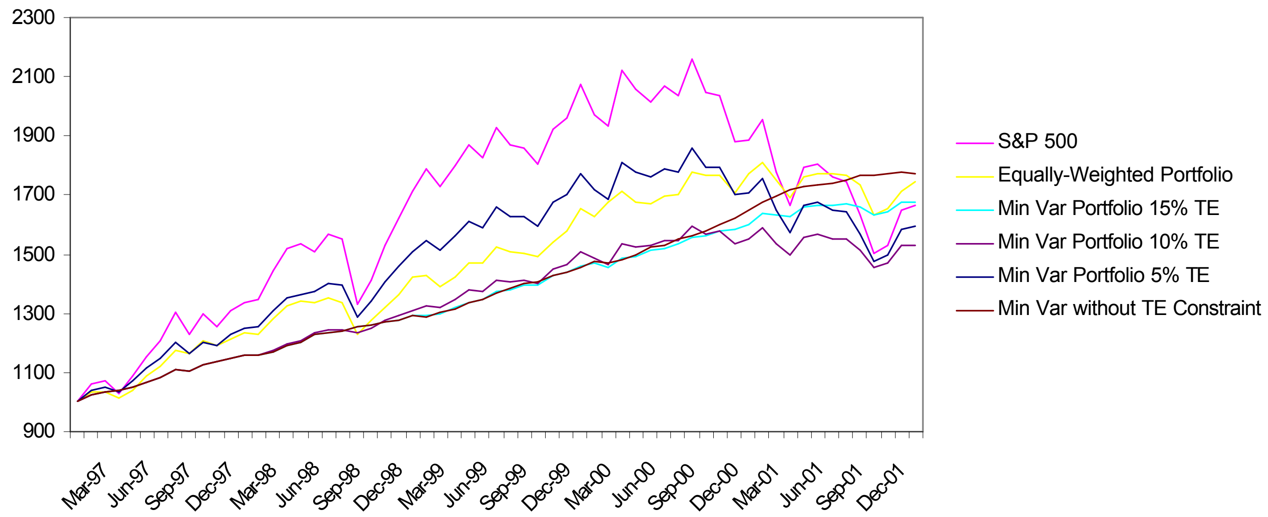

Figure 2: This graph displays the evolution of $1,000 invested in January 1997 in the S&P 500, an equally-weighted portfolio of S&P 500 growth, S&P 500 value, S&P 400 mid-cap and S&P 600 small-cap for the traditional part, and in Tremont dedicated short bias, Tremont market neutral and Tremont long/short, and the minimum variance portfolios obtained from an implicit factor-based variance-covariance matrix estimator, in the presence of a 5%, 10% and 15% tracking error constraints, as well as the minimum variance portfolio with no tracking error constraint.

As can be seen from the figure, minimum variance portfolios have much smoother paths than their equally-weighted and value-weighted counterparts. This is of course particularly true in the case of loose 15% tracking error constraint, or when the tracking error constraint is relaxed.

4 Conclusion

This paper is perhaps the first to evaluate the out-of-sample performance of an improved estimator of the covariance structure of hedge fund index returns, focusing on its use for optimal portfolio selection. Because of the presence of large estimation risk in the estimated expected returns, we choose to focus on the minimum variance portfolio of hedge fund indices. Using data from CSFB-Tremont hedge fund indices, we find that ex-post volatility of minimum variance portfolios generated using implicit factor based estimation techniques is between 1.5 and 6 times lower than that of a value-weighted benchmark (S&P 500), such differences being both economically and statistically significant. This strongly indicates that optimal inclusion of hedge funds in an investor portfolio can potentially generate a dramatic decrease in the portfolio volatility on an out-of-sample basis. Differences in mean returns, on the other hand, are not statistically significant, suggesting that the improvement in terms of risk control does not necessarily come at the cost of lower expected returns.[18]

Several other issues need to be addressed before the methodology can be fully implemented in practice, the first of which is the presence of transaction costs and other frictions forms of friction specific to alternative investments vehicles, such as long lockup periods. It would also be interesting to investigate the impact of constraints based on measures of extreme risk, such as VaR constraints. This is left for further research.

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[18] Similar results, that we do not report here, have been obtained for fixed-income oriented portfolios mixing alternative and traditional investment vehicles.

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6 Appendix: Information on Hedge Fund Strategies

- Convertible Arbitrage. Attempts to exploit anomalies in prices of corporate securities that are convertible into common stocks (convertible bonds, warrants, convertible preferred stocks). Convertible bonds tends to be under-priced because of market segmentation; investors discount securities that are likely to change types: if issuer does well, convertible bond behaves like a stock; if issuer does poorly, convertible bond behaves like distressed debt. Managers typically buy (or sometimes sell) these securities and then hedge part of or all of associated risks by shorting the stock. Delta neutrality is often targeted. Over-hedging is appropriate when there is concern about default as the excess short position may partially hedge against a reduction in credit quality.

- Dedicated Short Bias. Sells securities short in anticipation of being able to re-buy them at a future date at a lower price due to the manager's assessment of the overvaluation of the securities, or the market, or in anticipation of earnings disappointments often due to accounting irregularities, new competition, change of management, etc. Often used as a hedge to offset long-only portfolios and by those who feel the market is approaching a bearish cycle.

- Emerging Markets. Invests in equity or debt of emerging (less mature) markets that tend to have higher inflation and volatile growth. Short selling is not permitted in many emerging markets, and, therefore, effective hedging is often not available, although Brady debt can be partially hedged via U.S. Treasury futures and currency markets.

- Long/Short Equity. Invests both in long and short equity portfolios generally in the same sectors of the market. Market risk is greatly reduced, but effective stock analysis and stock picking is essential to obtaining meaningful results. Leverage may be used to enhance returns. Usually low or no correlation to the market. Sometimes uses market index futures to hedge out systematic (market) risk. Relative benchmark index is usually T-bills.

- Equity Market Neutral. Hedge strategies that take long and short positions in such a way that the impact of the overall market is minimized. Market neutral can imply dollar neutral, beta neutral or both.

  - Dollar neutral strategy has zero net investment (i.e., equal dollar amounts in long and short positions).
  - Beta neutral strategy targets a zero total portfolio beta (i.e., the beta of the long side equals the beta of the short side). While dollar neutrality has the virtue of simplicity, beta neutrality better defines a strategy uncorrelated with the market return.

  Many practitioners of market-neutral long/short equity trading balance their longs and shorts in the same sector or industry. By being sector neutral, they avoid the risk of market swings affecting some industries or sectors differently than others.

- Event Driven : corporate transactions and special situations

  - Deal Arbitrage (long/short equity securities of companies involved in corporate transactions)
  - Bankruptcy/Distressed (long undervalued securities of companies usually in financial distress)
  - Multi-strategy (deals in both deal arbitrage and bankruptcy)

- Fixed-Income Arbitrage. Attempts to hedge out most interest rate risk by taking offsetting positions. May also use futures to hedge out interest rate risk.

- Global Macro. Aims to profit from changes in global economies, typically brought about by shifts in government policy that impact interest rates, in turn affecting currency, stock, and bond markets. Participates in all major markets – equities, bonds, currencies and commodities – though not always at the same time. Uses leverage and derivatives to accentuate the impact of market moves. Utilizes hedging, but the leveraged directional investments tend to make the largest impact on performance.

- Managed Futures. Opportunistically long and short multiple financial and/or non financial assets. Sub-indexes include Systematic (long or short markets based on trend-following or other quantitative analysis) and Discretionary (long or short markets based on qualitative/fundamental analysis often with technical input).