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PUBLIC EDUCATION EXPENDITURES, HUMAN CAPITAL INVESTMENT AND INTERGENERATIONAL MOBILITY: A TWO-STAGE EDUCATION MODEL

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ABSTRACT

We show in this paper that, depending on the initial distribution of material wealth and that of individuals' abilities, economies converge in the long run towards different proportions of the skilled workforce and different levels of average wealth. We also show that the growth process raises net economic mobility, the long-run proportion of the skilled population and the long-run levels of wealth held by both rich and poor dynasties. Unless the income tax rate is too high, the increase in total public funds is associated, in the long run, with higher net mobility, a larger fraction of the skilled workers and higher levels of wealth of all the dynasties. In addition, the reallocation of public expenditures from basic to advanced education can result in lower mobility, a lower long-run size of the skilled workforce, and a lower long-run level of wealth held by rich dynasties, if the transfer of resources comes at the expense of excessively lowering the quality of education at the basic schooling level.

Keywords: distribution of wealth and abilities, economic mobility, human capital investment, public education provision policies
JEL classification numbers: H52, I22, I28, O1, O15

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I. INTRODUCTION

This paper analyses the dynamical relation between educational investment, wealth inequality and intergenerational economic mobility in a context of hierarchy in human capital investment and the assumptions of credit-market imperfections and heterogeneity in individuals' abilities. It then examines how public education funding policies may influence the economy. In particular, we are interested in the implications of two policies: the increase in the educational budget via raising the income tax rate; and the reallocation of public funds across basic and advanced education while holding total budget fixed. Our study combines three existing strands of literature.

The first strand focuses on the relation between inequality, human capital investment, and growth. This relation has been particularly prominent in the credit-market imperfections theory, where it has been commonly shown that unequal distributions of income combined with credit-market imperfections are constraints to investment and growth. This kind of analysis was first formulated in Loury (1981), and recently developed in Galor and Zeira (1993), Banerjee and Newman (1993), Aghion and Bolton (1997) and Piketty (1997), among others.[1]

While the works mentioned previously have not studied intergenerational economic mobility, another strand of literature has recently focused on this issue in order to analyse the interactions between economic growth and economic mobility. For instance, Galor and Tsiddon (1997) studied the effect of technological progress on intergenerational mobility and wage inequality. Their main result is that in a period of major technological inventions, the return to ability increases and the relative importance of initial conditions declines, leading to higher mobility. Hence, inventions raise both inequality and mobility.

Owen and Weil (1998) provided another interesting example in their study of mobility in the presence of capital-market imperfections and heterogeneity in individuals' abilities. In this study, mobility increases as a result of changes in the wage structure that accompany economic growth. In particular, in contrast to Galor and Tsiddon (1997), the increases in the fraction of the labour force that is educated reduce the wage gap between educated and uneducated workers, thus raising the probability that the children of uneducated workers will be able to afford an education.

Maoz and Moav (1999) study the dynamics of inequality and mobility along the growth path under the assumptions of an imperfect credit market and individual heterogeneity. They show that mobility promotes

[1] For the empirical literature on the evidence of credit constraints, the reader can refer to the micro-level studies of Kane (1994), Dynarski (1999) and Ellwood and Kane (2000) or to the macro-level studies conducted by De Gregorio (1996), Li and Zou (1998), Flug *et al.* (1998), Checchi (2000), Clarke *et al.* (2003) and Ben Mimoun (2008).

economic growth via its effect on the accumulation and allocation of human capital. In turn, growth influences mobility via its effect on incentives to acquire education as well as on liquidity constraints that bind poor individuals. Hence, in the process of development, mobility increases and the distribution of education becomes better correlated with ability.

In the same line of research, Iyigun (1999) considered a model in which admission to schools is competitive and capital markets are perfect. The study shows that an increase in the fraction of educated parents has two offsetting effects. First, by increasing total output, it expands the supplies of educational services. This would make admissions to school less competitive and would increase economic mobility. Second, an increase in the fraction of educated parents implies that some members of the younger generation have greater academic potential. This would make admissions to school more competitive, lowering mobility.

The third strand of literature on which our model is based focuses on the implications of increasing public resources toward the education sector for human capital accumulation, inequality and growth. Most theoretical studies in this strand of literature are based on the idea that additional expenditures on education enhance human capital accumulation and economic growth, and reduce income inequality.[2] As far as human capital investment is assumed to be indivisible in these studies, the education sector has only one schooling level, and public expenditures are considered in their aggregated form. However, by focusing on the implications of the educational expenditures in their aggregated form, previous studies have left untreated the fundamental question of how different allocations of public funds across the successive schooling levels affect the economy. Tackling this issue is crucial because it may contribute to a better understanding of why, in spite of the continuous increments in the educational budgets of many developing countries, namely countries in Africa and Latin America, post-primary schooling enrolment rates are still very low and income inequality is very high. Gupta *et al.* (1997, 2002), Benedict (1997) and Birdsall (1999) are excellent examples providing evidence on such paradoxical associations.

Very few studies in recent years have emphasized the implications of the allocation of educational expenditures for the economy. For instance, Lloyd-Ellis (2000) shows – in the context of a two-stage education model – that a reallocation of expenditures from basic to higher education reduces enrolments in higher education and increases income inequality. Furthermore, the impact of the allocation of public resources on growth reflects a tension between the trickle-down effects of higher education and the positive enrolment effects of high-quality basic

[2] Some well-known examples are Glomm and Ravikumar (1992), Saint and Verdier (1993), Bénabou (1996) and Fernandez and Rogerson (1997, 1999).

education and reduced parental income inequality. Another interesting study with similar results was conducted by Xuejuan (2004). The author demonstrates that, since basic education is a prerequisite for attending advanced education, there exists a lower bound on funding basic education. It follows that allocation policies below this lower bound are strictly Pareto dominated. In addition, while an allocation policy favouring basic education generates the usual redistribution from top to bottom, a policy favouring advanced education may result in reverse redistribution from bottom to top.

The two studies discussed previously assume that capital markets are perfect, and therefore the schooling decisions are independent from the distribution of wealth. In addition, they have not explicitly considered the mobility issue. The analytical framework we develop in this paper fills these gaps. Credit markets are assumed imperfect, and the study of economic mobility is allowed by assuming heterogeneity in individuals' abilities and the possibility for some poor individuals to borrow. As in Lloyd-Ellis (2000) and Xuejuan (2004), we model human capital accumulation as a two-stage process, and not as one indivisible level. Indeed, one accurate interpretation of the low levels of average schooling of the working population observed in many countries is that a large fraction of this population does not acquire education beyond the primary level, which is compulsory in almost all countries.

We consider that all individuals must invest in the compulsory basic education (primary schooling), and should, at the end of this level, decide whether to acquire advanced education (secondary and higher education). Individuals base their decisions on the level of their ability endowments and their parental financial transfers. The analysis of the dynamics of wealth transfers shows that the distribution of abilities and that of initial wealth play a role in the acquisition of advanced education in the long run. This analysis also enables us to detail the possibilities of upward and downward economic mobility. We find that there is a possibility of multiple steady-state equilibria with different levels of investment in advanced education, mobility and average wealth; and the specific one the economy converges to depends on the distribution of initial wealth. Another crucial result that emerges from analysing the dynamics of the model concerns the evolution of the economy along the growth process. We show that, by raising public provisions allocated towards all the levels of education, the growth process fosters aggregate investment in the advanced level, raises net mobility and increases the long-run levels of wealth of all dynasties.

Concerning the implications of the public education funding policies we find that, unless the financing of the education budget is highly distortional, increasing the income tax rate affects positively the long-run size of the skilled population, economic mobility and the levels of wealth of both rich and poor dynasties. Furthermore, the effects of reallocating the public funds from basic to advanced education on the acquisition of advanced education depend on the interplay between two forces of opposite signs: the negative effect on the liquidity constraints for the poor, and the positive effect on the quality of education received at the advanced level for the rich. Therefore, we show that above a certain allocation of expenditures in favour of advanced education, additional transfers of public resources from basic to higher education result in the long run in a lower fraction of skilled population, lower net mobility and lower levels of wealth that are held by rich and poor dynasties.

This paper is organized as follows. In Section II, the analytical model is presented and the optimal individual's behaviours are discussed. Section III analyses the dynamics of wealth transfers and examines the possibilities of intergenerational economic mobility. Section IV extends the dynamic analysis to the study of the evolution of the economy along the growth process. In Section V, the implications of the education provision policies for the economy are studied in both the short run and the long run.

II. A TWO-STAGE EDUCATION MODEL

II.1 Description of the economy

II.1.1 The households. Consider overlapping generations with heterogeneous individuals. Individuals in each generation differ in two respects: they inherit different financial supports from their parents and have different talents (or abilities to benefit from education). Financial inheritances are noted by $x \in [\underline{x}, \bar{x}]$ with the density function $f(x)$. Abilities noted by a evolve in the interval $[\underline{a}, \bar{a}]$ and are assumed to have an exogenous probability density function, $g(a)$. For tractability of the analysis, ability endowments are defined as the set of talents that individuals are born with and are therefore assumed to be distributed independently from parental wealth.[3] We use the subscript t in the model to index the generations. Each generation lives for three periods, during which individuals invest in education and work.

Education is accumulated in a hierarchical way. We model this hierarchy as a two-stage dependent process. In the first period, all individuals are enrolled in the compulsory basic education. In the second period, the human capital stock from basic education is used as an input for

[3] While one can argue that abilities are not strictly and independently distributed from wealth, one can agree that the inherent association, if any, is not strong. In fact, although the material wealth one is born with has a determining effect on how one's abilities are developed and how successful one is later in life, it is not always true that the level of abilities one is endowed with at an early age is conditional on the parental material wealth, and vice versa.

the accumulation of advanced education. Less able individuals cannot benefit from advanced education even if they are born to rich parents. These individuals join the labour market and work during their second and third periods as unskilled workers. Only individuals with both sufficient abilities and parental financial support can invest in advanced education. These individuals work in their third period of life as skilled workers. In addition to public expenditures, investment in advanced education involves a 'private' cost, which is assumed to be fixed at ϕ for all individuals. Individuals consume in the third period only. At the end of life, each individual is replaced by one offspring, such that the population remains constant. The size of each generation is assumed to be unity. In the Appendix Table A clarifies what activities are taking place during the three stages of the agents' lives.

Let h_{Bt} and h_{At} note the unskilled and skilled workers' human capital stocks (or incomes), respectively. Indexes B and A refer, respectively, to basic and advanced educational levels. The stock of basic education depends on the level of the individual's ability and the quality of public education received at this stage. In turn, the basic human capital stock and the quality received at the advanced schooling level are inputs in the accumulation function of advanced human capital. We formally assume the following relations:

$$\begin{cases} h_{Bt} = h_{Bt}(a, E_{Bt}) = aE_{Bt}^{\alpha} \\ h_{At} = h_{At}(h_{Bt}, E_{At}) = h_{Bt}E_{At}^{\gamma} \end{cases} \tag{1}$$

where a represents the individual's ability and E_{Bt} and E_{At} are, respectively, the quality of public education at the basic and advanced educational stages. This quality is simply proxied by the amount of public resources invested in each schooling level. The parameters α and γ are in the $[0, 1]$ interval.

The assumed functional form captures one key characteristic of the production function of human capital: there are complementarities between the ability effect and public expenditures (i.e., $\partial^2 h_j/\partial a \partial E_j > 0 \, \forall j$, $j = B, A$). Such complementarities assumption is consistent with the formulation presented in Lucas (1988), Bénabou (1996), Loury (1981), Pinera and Selowsky (1981), Saint and Verdier (1993), Glomm and Ravikumar (1992) and Glomm and Kaganovich (2003). However, by contrast to these studies where the quality of education is assumed to be the same for all students, our model suggests that this quality differs with respect to the educational stage.

Individuals derive utility both from consumption and from bequests to their offspring. That is, there is intergenerational altruism taking the form of parents having the joy of giving to their offspring. The following utility function is assumed:

$$V_t = \rho \log C_t + (1 - \rho) \log x_{t+1} \tag{2}$$

where C_t is consumption of the generation t and x_{t+1} is the parent's bequest to his child. $1 - \rho$ denotes the importance of the bequest in the utility function. Individuals' lifetime wealth is allocated between own consumption and bequest to the offspring.

II.1.2 The government. It is assumed that the government collects tax revenue from one generation, and allocates the public funds between the basic and advanced stages of education for the next generation. If we note by Y_{t-1} the aggregated income of parents and by τ the income tax, then total expenditures are τY_{t-1}. The shares of expenditures allocated to basic and advanced education are constant and are given by e_B and $1 - e_B$, respectively. Hence, the quality of education at the basic level may be formulated as follows:

$$E_{Bt} = e_B \tau \; Y_{t-1} \tag{3}$$

At the advanced level, the quality of education is given by

$$E_{At} = (1 - e_B)\tau Y_{t-1} \tag{4}$$

II.1.3 The credit market. There are several ways to model credit-market imperfections. Either credit markets can be considered as completely absent (the extreme case), or individuals should be sufficiently endowed with initial wealth to borrow. Eventually, individuals can obtain credit, but have to pay an interest rate that covers the lender's interest rate and the borrower's cost of possible default.

We adopt the last form of imperfections as in Galor and Zeira's (1993) model. The economy we consider is small and open to the world capital market. The world rate of interest is equal to $r > 0$ and is assumed to be constant over time. Borrowers have the possibility to evade debt payments by moving to other places and so on, but this activity is costly. Lenders can avoid defaults by keeping track of borrowers, but such precautionary measures are also costly. The borrower's cost of evasion is assumed to be higher than the lender's cost of keeping track of borrowers. These costs create capital-market imperfections, so that individuals can borrow only at an interest rate i, which is higher than r, the lender's interest rate (i.e., $i > r$). Such imperfections make borrowing costly, and may prevent some poor individuals, although with high abilities, from borrowing.[4]

II.1.4 Definition of equilibrium. Given a density function of wealth $f_t(x)$, a density function of individuals' abilities $g(a)$, exogenous

[4] Galor and Zeira (1993) argue that under any other specification of credit-market imperfections, as long as borrowing is not fully free and costless, those who inherit large amounts have easier access to investment in human capital than those with small bequests.

parameters of public policy and credit market (τ, e_B, r, i) and the cost of education ϕ, a period t equilibrium is defined as a vector $\{C_t^*, x_{t+1}^*, S_t\}$ so that

- the government balances its budget, $E_{Bt} + E_{At} = \tau Y_{t-1}$;
- individuals determine their consumption, C_t^*, and the bequests to their offspring, x_{t+1}^*, that maximize utility (Equation (2)) subject to Equations (1), (3) and (4);
- individuals' decisions whether to invest in advanced education determine the fraction of skilled individuals in period t, S_t.

II.2 Optimal behaviour

Consider an individual who inherits an amount x_t in the first period of life. We should distinguish three types of decisions.

1. If $(1 + r)x_t < \phi$, and the individual does not invest in advanced education, he will be an unskilled worker with a lifetime utility given by

$$V_{Bt} = \log[(1 - \tau)(2 + r)h_{Bt}(a) + x_t(1 + r)] + \xi \tag{5}$$

where $\xi = \rho \log \rho + (1 - \rho) \log(1 - \rho)$. This worker has a consumption of

$$C_{Bt}(x_t, a) = \rho[(1 - \tau)(2 + r)h_{Bt}(a) + x_t(1 + r)] \tag{5a}$$

He will leave a bequest of size

$$B_{Bt}(x_t, a) = x_{t+1} = (1 - \rho)[(1 - \tau)(2 + r)h_{Bt}(a) + x_t(1 + r)] \tag{5b}$$

2. If $(1 + r)x_t < \phi$, and the individual decides to invest in advanced education, he is a borrower and will be a skilled worker in his last period of life. His lifetime utility is

$$V_{At} = \log\{(1 - \tau)h_{At}(a) + (1 + i)[(1 + r)x_t - \phi]\} + \xi \tag{6}$$

This worker has a consumption of

$$C_{At}(x_t, a) = \rho\{(1 - \tau)h_{At}(a) + (1 + i)[(1 + r)x_t - \phi]\} \tag{6a}$$

He will leave a bequest of

$$B_{At}(x_t, a) = x_{t+1} = (1 - \rho)\{(1 - \tau) h_{At}(a) + (1 + i)[(1 + r)x_t - \phi]\} \tag{6b}$$

3. If $(1 + r)x_t \geq \phi$, and the individual decides to invest in advanced education, he is a lender and will be a skilled worker with a lifetime utility of

$$V_{At} = \log\{(1 - \tau)h_{At}(a) + (1 + r)[(1 + r)x_t - \phi]\} + \xi \tag{7}$$

He has a consumption of

$$C_{At}(x_t, a) = \rho\{(1-\tau)h_{At}(a) + (1+r)[(1+r)x_t - \phi]\} \quad (7a)$$

He will leave a bequest of

$$B_{At}(x_t, a) = x_{t+1} = (1-\rho)\{(1-\tau)h_{At}(a) + (1+r)[(1+r)x_t - \phi]\} \quad (7b)$$

One can deduce from Equations (5) and (6) that borrowers invest in advanced education as long as $V_{At} \geq V_{Bt}$. Using the relations in Equation (1), this condition yields the following threshold level of financial wealth

$$x_t^*(a) = \frac{(1+i)\phi - (1-\tau)\, aE_{Bt}^{\alpha}\left[E_{At}^{\gamma} - (2+r)\right]}{(1+r)(i-r)} \quad (8)$$

The fact that this threshold depends on a implies that there is a critical level of financial wealth for each level of ability. One can easily point out that the higher the individual's ability, the lower is the critical wealth level of that individual. Furthermore, for a given level of ability, this threshold is increasing in the private cost of education, ϕ, and decreasing in public expenditures that are invested in both stages of education.

Lenders decide to invest in advanced education as far as their lifetime utility is higher than that of the unskilled workers. This holds only for lenders that are endowed with at least an ability of

$$a^* = \frac{\phi(1+r)}{(1-\tau)E_{Bt}^{\alpha}\left[E_{At}^{\gamma} - (2+r)\right]} \quad (9)$$

Hence, financial and ability thresholds expressed in Equations (8) and (9) determine the fraction of individuals that would invest in advanced education, in period t. This fraction is given as follows:

$$S_t = \int_{a^*}^{\bar{a}} \int_{x_t^*(a)}^{\bar{x}} f_t(x_t)\, g(a)\, \mathrm{d}x\, \mathrm{d}a \quad (10)$$

Thus, in the short run, the size of the skilled population is a function of the distribution of individuals' abilities, and of the initial distribution of wealth (i.e., in $t = 0$), since the fraction of individuals that invests in advanced education is determined by the proportion of the population that has inherited more than $x_t^*(a)$ in period t, and is at the same time endowed with abilities more than a^*. We show subsequently that the initial distribution of wealth also determines the size of the skilled workers in the long run.

III. THE DYNAMICS OF DYNASTIES AND INTERGENERATIONAL MOBILITY

III.1 Evolution of dynasties

This section derives the dynamics of wealth transmission and determines the long-run proportion of skilled workers as well as the distribution of wealth across rich and poor dynasties. The bequest an individual gives to his offspring depends on that individual's inheritance and his labour income, with labour income depending on ability. Hence, the distributions of both inheritances and abilities in period t determine the distribution of bequests in period $t + 1$. According to Equations (5b), (6b) and (7b), these dynamics can be presented as follows:

$$x_{t+1} = \begin{cases} B_{Bt}(x_t, a) = (1-\rho)[(1-\tau)(2+r)h_{Bt}(a) + x_t(1+r)] \\ \qquad \text{if } x_t < x_t^* \text{ or } a < a^* \\ B_{At}(x_t, a) = (1-\rho)\{(1-\tau)h_{At}(a) + (1+i)[(1+r)x_t - \phi]\} \\ \qquad \text{if } x_t^* \leq x_t < \phi/(1+r) \text{ and } a \geq a^* \\ B_{At}(x_t, a) = (1-\rho)\{(1-\tau)h_{At}(a) + (1+r)[(1+r)x_t - \phi]\} \\ \qquad \text{if } \phi/(1+r) \leq x_t \text{ and } a \geq a^* \end{cases} \tag{11}$$

where x_0 is given.

Recall that B_{Bt} is the financial bequest of unskilled workers (those with only basic education), and B_{At} is that of skilled workers (both borrowers and lenders with advanced education). System (11) defines a Markov process where the size of a bequest, x_{t+1}, is conditional on the size of inheritance, x_t, and the level of abilities, a. The first equation of the system implies that individuals with either very low inheritance or low ability would transfer B_{Bt} to their children, as they are excluded from investing in advanced education. The last two equations point out that those having inherited more than $x_t^*(a)$ must also be endowed with abilities higher than a^*, in order to transfer to their children a bequest of B_{At}.

Figure 1 illustrates the dynamical relationship between inheritances and bequests for both poor and rich dynasties, while considering the case of $a = \underline{a}$ for the group of individuals with abilities ranging between $\underline{a}$ and a^*, and the case of $a = \bar{a}$ for those with abilities between a^* and $\bar{a}$.

Notice that we impose the condition that $(1-\rho)(1+r) < 1$, so that the size of a transfer does not grow indefinitely. An additional assumption, which is implicit in Figure 1, is that $(1-\rho)(1+i)(1+r) > 1$. That is, the cost of keeping track of borrowers is high, so that the spread between the lending and borrowing interest rates is high as well.

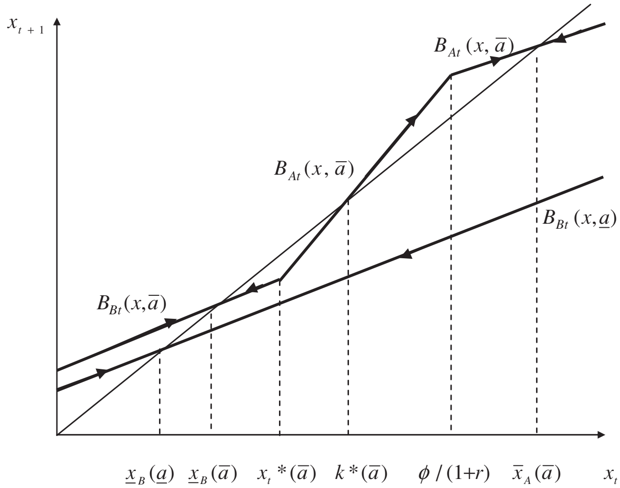

Fig. 1. The dynamics of intergenerational wealth transmission.

The dynamics of wealth transmission can be understood as the following.

1. Independently of their initial wealth, all individuals with ability $a < a^*$ cannot go beyond basic education, and are therefore employed as unskilled workers. Their bequests are represented by the straight line, B_{Bt}. For this group, the example of $a = \underline{a}$ is considered for graphical representation. An increase in a shifts up the locus $B_{Bt}(x, \underline{a})$. Inheritances of these individuals converge in the steady state to the lower long-run values $\underline{x}_B(a)_{a<a^*}$ given by

$$\underline{x}_B(a)_{a<a^*} = \frac{(1-\rho)(1-\tau)(2+r)E^{\alpha}_{Bt}a}{1-(1-\rho)(1+r)^2} \tag{12}$$

Indeed, individuals in this range of abilities, who received a transfer of less than $\underline{x}_B(a)_{a<a^*}$ pass on to their children a transfer larger than the one they received. Those having received a transfer of more than $\underline{x}_B(a)_{a<a^*}$ pass on to their children a transfer that is less than the one they received.

2. Individuals with $a \geq a^*$, who inherited more than $x^*_t(a)_{a>a^*}$, invest in higher education, but not all of their descendants remain in the skilled labour sector. The critical wealth levels are $k^*(a)_{a\geq a^*}$, where

$$k^*(a)_{a\geq a^*} = \frac{(1-\rho)\left[(1+i)\phi-(1-\tau)E^{\alpha}_{Bt}E^{\gamma}_{At}a\right]}{(1-\rho)(1+i)(1+r)-1} \tag{13}$$

For this range of abilities, Figure 1 considers the example of individuals endowed with ability of $a = \bar{a}$. The critical wealth level in this case is given by $k^*(\bar{a})$. In the case of these individuals, three configurations can be followed.

- Individuals with $a \geq a^*$, who inherited less than $k^*(a)_{a \geq a^*}$ in period t, pass on to their children bequests that are less than the ones they received. Therefore, these individuals may work as skilled workers (since they inherited more than $x^*(a)$), but after some generations their descendants become unskilled workers, and their inheritances converge to $\underline{x}_B(a)_{a \geq a^*}$. In Figure 1, this is represented by the point $\underline{x}_B(\bar{a})$, for the case of $a = \bar{a}$.
- However, individuals with $a \geq a^*$ who inherited more than $k^*(a)_{a \geq a^*}$ would bequeath values higher than the ones they received. In the long run, their bequests converge to the highest values $\bar{x}_A(a)_{a \geq a^*}$ given by

$$\bar{x}_A(a)_{a \geq a^*} = \frac{(1-\rho)\left[(1-\tau)E_{Bt}^{\alpha}E_{At}^{\gamma}a - (1+r)\phi\right]}{1-(1-\rho)(1+r)^2} \quad (14)$$

  Figure 1 considers the case of individuals that are endowed with $a = \bar{a}$, and shows that the wealth of those individuals converge, in the long run, to the point $\bar{x}_A(\bar{a})$.
- Individuals with $a \geq a^*$ who inherit more than $\phi/(1+r)$ invest in higher education. They remain in the skilled labour sector, generation after generation, and their bequests converge to the highest long-run levels given by $\bar{x}_A(a)_{a \geq a^*}$.

To sum up, the population in this economy is divided in two groups in the long run: skilled workers and unskilled workers. Skilled workers have a wealth of $\bar{x}_A(a)$, whereas unskilled workers have a wealth of $\underline{x}_B(a)$, with both wealth levels increasing in the individuals' abilities. The relative size of these two groups depends unambiguously on the initial distribution of wealth, as well as on the distribution of abilities. Indeed, in the long run, the proportion of the highly educated population, noted below by $\tilde{S}$, is determined by the individuals who inherited more than $k^*(a)$ in period t and have, at the same time, more than a^*. That is,

$$\tilde{S} = \int_{a^*}^{\bar{a}} \int_{k^*(a)}^{\bar{x}} f_t(x_t)g(a)\mathrm{d}x\,\mathrm{d}a \quad (15)$$

In what follows, we study the different possibilities of interclass mobility across generations, and confirm that the fraction of rich dynasties is given, in the long run, by the fraction of individuals with advanced education, $\tilde{S}$.

TABLE 1
The transition probability matrix

	Child type	
	Rich	*Poor*
Parent type		
Rich	$\Pr(r/r)$	$\Pr(p/r)$
Poor	$\Pr(r/p)$	$\Pr(p/p)$

III.2 Intergenerational economic mobility

We define economic mobility as the change in dynasties' adherence to income groups between generations. *Upward mobility* refers to a situation in which individuals although born to poor parents (i.e., with $x_t < \phi/(1+r)$), acquire advanced education and become rich. *Downward mobility* refers to a situation in which individuals born to rich parents (i.e., with $x_t \geq \phi/(1+r)$), do not invest in advanced education and become poor. Finally, the *no mobility* case is the situation in which children whose parents are rich also become rich, and children whose parents are poor remain poor.

Downward mobility arises in our model as some individuals born to rich parents do not acquire advanced education because of their low levels of ability (i.e., $a < a^*$).

Upward mobility, however, concerns the fraction of individuals with inheritance of $x_t \in [k^*(a)_{a \geq a^*}, \phi/(1+r)]$. It occurs because individuals who inherit more than $k^*(a)_{a \geq a^*}$ would bequeath values higher than the ones they received, which allows their offspring to be skilled workers, generation after generation. The possibility of upward mobility for these individuals is strengthened because individuals with high levels of abilities have lower levels of wealth thresholds above which they become highly educated. Finally, no mobility concerns all dynasties that are either both rich and highly talented or with wealth less than $k^*(a)$. One possible way to measure economic mobility is by means of a transition probability matrix, as shown in Table 1, where

- $\Pr(r/r)$ is the probability that children born to rich parents remain rich (or, equivalently, the fraction of rich individuals born to rich parents), which is given by

$$\Pr(r/r) = \int_{a^*}^{\bar{a}} \int_{\phi/(1+r)}^{\bar{x}} f_t(x_t)\, g(a)\, \mathrm{d}x\, \mathrm{d}a$$
$$= \{1 - F_t[\phi/(1+r)]\}[1 - G(a^*)]$$

- $\Pr(p/r)$ is the probability that children born to rich parents become poor (or, equivalently, the fraction of poor individuals born to rich parents), and is given by

$$\Pr(p/r) = \int_{\underline{a}}^{a^*} \int_{\phi/(1+r)}^{\bar{x}} f_t(x_t)\, g(a)\, \mathrm{d}x\, \mathrm{d}a = \{1 - F_t[\phi/(1+r)]\} G(a^*)$$

- $\Pr(r/p)$ is the probability that children born to poor parents become rich (or, equivalently, the fraction of rich individuals born to poor parents), and is written as

$$\Pr(r/p) = \int_{a^*}^{\bar{a}} \int_{k^*(a)}^{\phi/(1+r)} f_t(x_t)\, g(a)\, \mathrm{d}x\, \mathrm{d}a$$

- $\Pr(p/p)$ is the probability that children born to poor parents remain poor (or, equivalently, the fraction of poor individuals born to poor parents), and is defined by

$$\Pr(p/p) = \int_{\underline{a}}^{a^*} \int_{k^*(a)}^{\phi/(1+r)} f_t(x_t)\, g(a)\, \mathrm{d}x\, \mathrm{d}a + \int_{\underline{a}}^{\bar{a}} \int_{\underline{x}}^{k^*(a)} f_t(x_t)\, g(a)\, \mathrm{d}x\, \mathrm{d}a$$

  Notice that $F_t(.)$ and $G(.)$ are, respectively, the distribution functions of $f_t(.)$ and $g(.)$, and that the sum of these probabilities is unity.

It follows from these probabilities that the proportions of upwardly and downwardly mobile individuals are given by $\Pr(r/p)$ and $\Pr(p/r)$ respectively.

By referring to the expression of each of these probabilities given previously, one can unambiguously show that downward mobility increases in $G(a^*)$, which is the fraction of individuals that are endowed with ability less than a^*, and that upward mobility increases in the fraction of the population with more than both a^* and $k^*(a)$.

If we note the fraction of rich individuals in period t as $R_t = 1 - F_t[\phi/(1+r)]$, it follows that the fraction of rich individuals in $t+1$, R_{t+1}, is higher than R_t as long as upward mobility exceeds downward mobility, and vice versa. As the fractions of upwardly and downwardly mobile individuals are equal (i.e., $\Pr(r/p) = \Pr(p/r)$), R_t reaches its long-run equilibrium value, noted by $\tilde{R}$, where

$$\tilde{R} = \int_{a^*}^{\bar{a}} \int_{k^*(a)}^{\bar{x}} f_t(x) g(a) \mathrm{d}x_t\, \mathrm{d}a = \tilde{S} \tag{16}$$

$\tilde{S}$ is given in Equation (15), and denotes the long-run fraction of individuals that invest in advanced education (or the skilled population). This fraction is a function of the initial distribution of wealth as well as the distribution of abilities.

As the long-run fractions of rich and poor dynasties as well as their corresponding levels of wealth are determined, the long-run aggregate (or average) wealth of the economy – noted below by $\tilde{X}$ – can be defined as follows:

$$\tilde{X} = \int_{a^*}^{\bar{a}} \int_{k^*(a)}^{\bar{x}} \bar{x}_A(a)\, f_t(x)\, g(a)\, \mathrm{d}x_t\, \mathrm{d}a + \int_{\underline{a}}^{a^*} \int_{\underline{x}}^{\bar{x}} \underline{x}_B(a)\, f_t(x)\, g(a)\, \mathrm{d}x_t\, \mathrm{d}a + \int_{a^*}^{\bar{a}} \int_{\underline{x}}^{k^*(a)} \underline{x}_B(a)\, f_t(x)\, g(a)\, \mathrm{d}x_t\, \mathrm{d}a \tag{17}$$

The first term on the right-hand side of Equation (17) corresponds to the long-run share of wealth held by the rich population, while the second and third terms represent the long-run wealth of the poor population. Clearly, $\tilde{X}$ increases in the fraction of rich population, $\tilde{R}$, and is consequently positively correlated with the proportion of the population that is initially endowed with a wealth more than $k^*(a)$ and with abilities more than a^*.

To sum up the results established in this section, one can assess that economies with identical taste and technology parameters, but different initial wealth distributions, can end up in different steady states of investment in advanced education, mobility and average wealth. The country with a more equal initial wealth distribution will have higher steady-state levels. That is, there are multiple long-run equilibria and the specific one the economy converges to depends on the initial distribution of wealth.

Proposition 1. The economy's long-run levels of investment in advanced education, mobility and aggregate (average) wealth depend on the initial distribution of wealth.

IV. THE EVOLUTION OF THE ECONOMY ALONG THE GROWTH PROCESS

This section analyses the changes in mobility and in the distribution of wealth along the growth process. The growth process can be emphasized in the model as an increase in aggregate (average) income, Y_{t-1}. As shown in Equations (3) and (4), an increase in aggregate income expands the supply of educational expenditures in both basic and advanced schooling levels, leading to an improvement in the quality of education at both levels.

In the short run, such improvement has two positive reinforcing effects on the fraction of individuals with advanced education, S_t. On the one hand, liquidity constraints on the poor are relaxed as human capital (or income) of those with basic education increases. On the other hand, incentives for investment in advanced education increase among the rich as their incomes increase too. Accordingly, the proportion of individuals who afford advanced education, S_t, rises as the economy's total income increases. In order to illustrate this result, one may easily check that both ability and wealth thresholds, a^* and $x_t^*(a)_{a \geq a^*}$ respectively, are monotonically decreasing in Y_{t-1}.

In the long run, the growth process raises the fraction of rich individuals – or equivalently that of highly educated workers – because net mobility is increased. Indeed, since the thresholds of a^* and $k^*(a)_{a \geq a^*}$ are both monotonically decreasing in Y_{t-1}, upward mobility rises and downward mobility falls. Hence, net economic mobility increases along the growth path. As a result, the fraction of rich individuals in the long run, $\tilde{R}$, goes up as is illustrated by Equation (16).

Furthermore, as shown in Equations (12) and (14), the growth process also results in higher long-run values of wealth held by both poor and rich dynasties (i.e., $\underline{x}_B(a)_{a<a^*}$, $\underline{x}_B(a)_{a \geq a^*}$ and $\bar{x}_A(a)_{a \geq a^*}$ are increasing in Y_{t-1}). Therefore, the long-run aggregate (average) wealth, $\tilde{X}$, increases along the growth path. These results are summarized in the following proposition.

Proposition 2. Along the growth process, both investment in advanced education and mobility are increased. In the long run, the fraction of rich individuals (or equivalently, of highly educated workers) as well as the levels of wealth held by both rich and poor dynasties are raised up.

V. EDUCATIONAL EXPENDITURE POLICIES

In this section, we explore the impacts of educational provision policies on investment in advanced education, economic mobility and the distribution of wealth. Two educational funding policies are examined. The first is an increase in total public education expenditures, which is financed by an increase in the tax rate, τ. The second policy is a reallocation of these resources across the two levels of education while holding the tax rate fixed. More specifically, this policy consists of varying the share of expenditures allocated to basic education, e_B. We show subsequently that how public funds are allocated across the two levels has direct implications on investment in advanced education and, consequently, on the aggregate economy. Throughout this analysis, policy implications are examined in both the short run and the long run.

V.1 First policy: an increase in the total budget for education

Under this policy, it is assumed that the shares of expenditures allocated to basic and advanced education stages are fixed (i.e., e_B is given). The government may increase the total budget for education by increasing the tax rate, τ. This policy has both short-run and long-run effects.

V.1.1 The short-run effects. There are two opposite effects in the short run associated with the increase in the tax rate. First, as shown by Equations (3) and (4), this policy simultaneously improves the quality of education at both basic and advanced stages. As a result, the stock of human capital accumulated by students with basic education, h_{Bt}, increases, implying a relaxation in the liquidity constraints that face the poor. At the same time, the income of highly educated individuals, h_{At}, increases, implying higher incentives for the rich to acquire advanced education. Second, because the increase in the education budget is financed through distortional income taxation, the higher the tax rate, the lower is the disposable income of both skilled and unskilled individuals. This distortion effect tends to reduce the incentives to acquire education. Nevertheless, this negative effect is always outweighed by the positive effect of increasing incentives, so that the fraction of individuals that invests in advanced education, S_t, monotonically increases with the income tax rate. To illustrate this result, one may see that both ability and wealth thresholds, a^* and $x_t^*(a)$ respectively, are monotonically decreasing in the tax rate, τ (proofs are in the Appendix). This result is summarized in the following proposition.

Proposition 3. In the short run, the fraction of individuals that invests in advanced education increases in a monotonic way with respect to the tax rate, τ.

V.1.2 The long-run effects. Varying the education budget through income taxation affects the fractions of upwardly and downwardly mobile individuals. This, consequently, influences the long-run proportion of rich individuals $\tilde{R}$ (or, equivalently, the fraction of highly educated population, $\tilde{S}$).

- It is worthwhile noticing that, like the threshold of ability a^*, the fraction of downwardly mobile individuals is monotonically decreasing with the tax rate τ.
- Upward mobility, however, depends on the thresholds of both ability, a^*, and wealth, $k^*(a)$. By using Equation (13), it is easy to show that, for any level of ability, the threshold $k^*(a)$ decreases (increases) in

the tax rate τ if $\tau < \tau^*$ ($\tau > \tau^*$), where

$$\tau^* = \frac{\alpha + \gamma}{1 + \alpha + \gamma} \tag{18}$$

Hence, as long as $\tau \leq \tau^*$, the fraction of upwardly mobile individuals increases with τ because both a^* and $k^*(a)$ are decreasing in τ. Conversely, when $\tau > \tau^*$, a^* is decreasing in τ and $k^*(a)$ is increasing. Thus, upward mobility may increase or decrease depending on the magnitude of the impacts of the tax rate on these thresholds.

Indeed, two cases are possible when $\tau > \tau^*$.

- Case (a): if $\partial k^*/\partial \tau < \partial a^*/\partial \tau$, then upward mobility increases with the tax rate, τ.
- Case (b): if $\partial k^*/\partial \tau > \partial a^*/\partial \tau$, then upward mobility decreases with the tax rate, τ.

The variations in upward and downward mobility affect the long-run proportion of rich individuals, $\tilde{R}$. Specifically, $\tilde{R}$ rises if upward mobility increases and downward mobility decreases, and vice versa. Figure 2 illustrates the effects of varying the income tax rate on upward and downward mobility, as well as on the resulting stationary proportion of rich individuals in the long run.

Proposition 4

1. *As long as $\tau \leq \tau^*$, net mobility increases and the long-run proportion of rich individuals, $\tilde{R}$, rises with τ.*
2. *If $\tau > \tau^*$, there are two configurations:*
   - *if $\partial k^*/\partial \tau < \partial a^*/\partial \tau$, then $\tilde{R}$ increases with τ.*
   - *if $\partial k^*/\partial \tau > \partial a^*/\partial \tau$, then the evolution of $\tilde{R}$ with respect to τ is indeterminate.*

Varying the level of the education budget affects not only the distribution of the population in the long run, $\tilde{R}$, but also the levels of wealth held by each dynasty of the population. This effect is non-monotonic because of the distortion effects of taxation associated with this policy. Indeed, one can show the following.

1. The highest levels of wealth, $\bar{x}_A(a)_{a \geq a^*}$, are increasing (decreasing) in τ if $\tau \leq \tau^*$ ($\tau > \tau^*$), where τ^* is defined in Equation (18).
2. Similarly, the lowest levels of wealth, $\underline{x}_B(a)_{a<a^*}$ and $\underline{x}_B(a)_{a \geq a^*}$, are increasing (decreasing) in τ if $\tau \leq \tau^{**}$ ($\tau > \tau^{**}$), where

$$\tau^{**} = \frac{\alpha}{1 + \gamma} < \tau^* \tag{19}$$

These effects are illustrated in Figure 3.

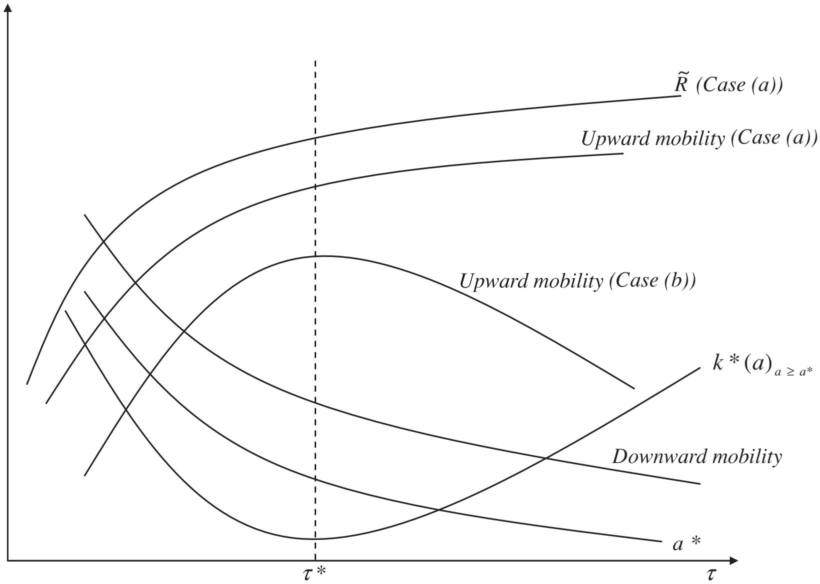

Fig. 2. The effects of the tax rate on mobility and the long-run size of rich dynasties.

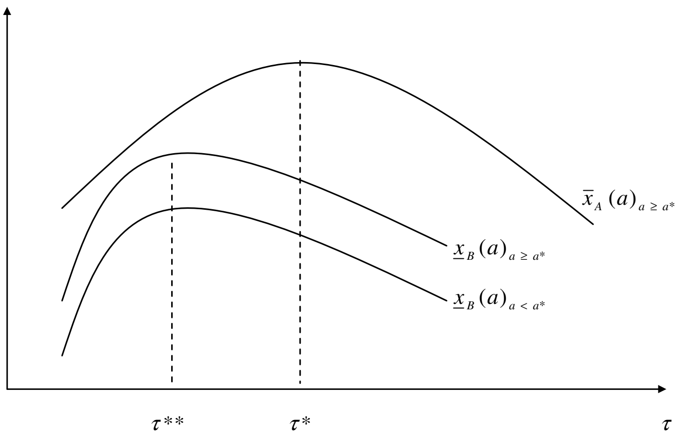

Fig. 3. The effects of the tax rate on rich and poor dynasties' long-run levels of wealth.

To summarize, as long as the income tax rate that finances the education-budget increments is not too high (i.e., $\tau \leq \tau^*$), the increase in the education budget is associated in the long run with a higher mobility, a higher proportion of rich population, and higher levels of wealth held by the rich and poor dynasties (the wealth of poor dynasties increases provided that τ is low enough). However, if the increase in the education budget is financed through highly distortional taxation (i.e., $\tau > \tau^*$), this policy decreases the long-run levels of wealth of both poor and rich dynasties, while its effect on the size of the rich population is ambiguous as it fosters both upward and downward mobility.

V.2 Second policy: the reallocation of expenditures across the stages of education

Under this policy scheme, the tax rate is fixed so that the total budget for education (τY_{t-1}) is fixed as well. The government affects the allocation of these resources across basic and advanced levels of education by varying e_B. This policy affects the economy in both the short run and the long run.

V.2.1 The short-run effects. How public expenditures are allocated across basic and advanced educational stages affects the number of students enrolled in the latter stage, S_t. Specifically, an increase in e_B improves the quality of basic education (i.e., E_{Bt} increases), but worsens the quality of advanced education (i.e., E_{At} decreases). Because of the hierarchical feature of educational investment, this policy implies two opposite effects on schooling decisions in the advanced stage. On one side, it increases the stock of human capital accumulated at the basic level, which, in turn, relaxes the liquidity constraints that face the poor and raises the fraction of students demonstrably able to continue investing in the advanced schooling level. On the other side, as the transfer of resources from advanced to basic education intensifies (i.e., when $e_B > 1/2$ in the case of $\alpha = \gamma = 1$), the associated reduction in the quality of education at the advanced level lowers the income of highly educated individuals, and therefore reduces the incentives for those individuals to invest in advanced education. Hence, investment in this level is governed by the interplay between these two effects.

To clarify this result, one may check that the ability and wealth thresholds, a^* and $x_t^*(a)$, evolve in a non-monotonic way with respect to the share of expenditures allocated to basic education, e_B.

In order to provide an analytical solution for the effect of varying e_B, let us consider the case of $\alpha = \gamma = 1$. It follows that both thresholds decrease (increase) in e_B if $e_B < e_B^*$ ($e_B > e_B^*$), where

$$e_B^* = \frac{\tau Y_{t-1} - (2 + r)}{2\tau Y_{t-1}} \tag{20}$$

This result is summarized in the following proposition.

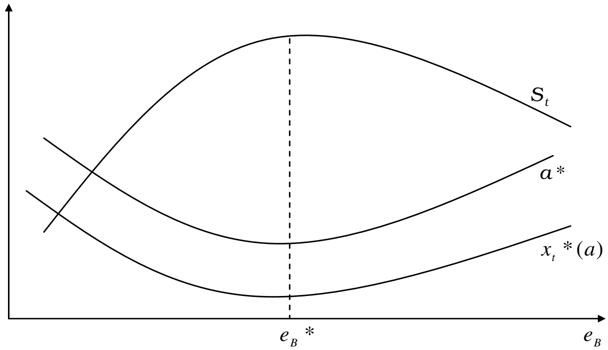

Fig. 4. The evolution of the skilled population with respect to the share of expenditures devoted to basic education.

Proposition 5. In the short run, given a fixed size of public education funds, an increase in e_B raises (decreases) the fraction of individuals that invests in advanced education if $e_B < e_B^*$ ($e_B > e_B^*$).

This relationship is illustrated in Figure 4. It shows that the size of the skilled population, S_t, can be increased when the expenditure on advanced education is decreased if the sums taken from the expenditure on this schooling level are transferred to basic education. Indeed, this transfer improves the quality of education at the basic level and raises the stock of human capital accumulated at this level. This in turn allows some individuals – namely those with high abilities – to invest in the advanced level. Nevertheless, the transfer of public resources toward basic education may discourage investment in advanced education if this transfer becomes excessive (i.e., if $e_B > e_B^*$).

V.2.2 The long-run effects. We show in this paragraph that, through its effect on individuals' mobility, the allocation of expenditures across the various stages of education affects the fraction of skilled individuals as well in the long run. This policy also alters the long-run levels of wealth held by the rich and the poor. The effects of the reallocation policy on upward and downward mobility are non-monotonic. Indeed,

1. as has shown in the previous paragraph, the ability threshold a^* and thus the fraction of downwardly mobile individuals is decreasing (increasing) in e_B if $e_B < e_B^*$ ($e_B > e_B^*$), where e_B^* is given in Equation (20);
2. the effects of varying e_B on both ability and wealth thresholds, a^* and $k^*(a)$ respectively, determine how upward mobility evolves with respect to e_B. It can be shown in the case of $\alpha = \gamma = 1$ that the

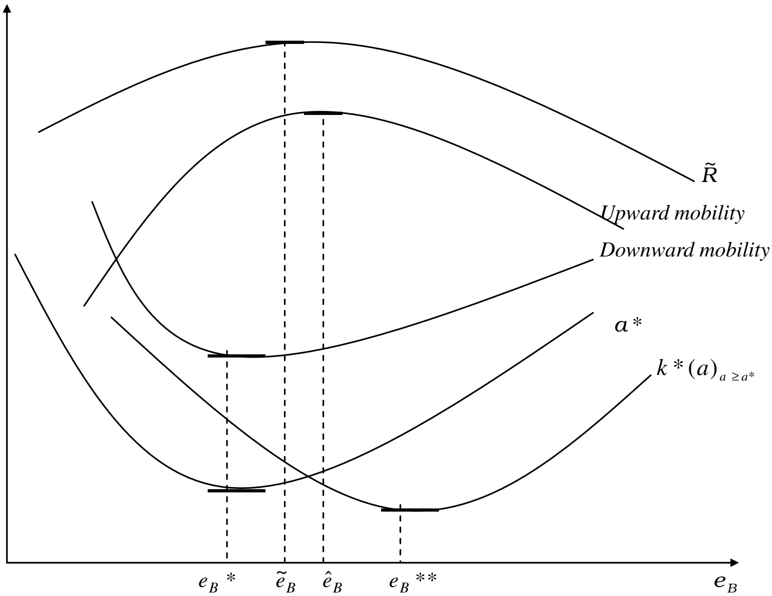

Fig. 5. The effects of expenditure allocation on mobility and the long-run size of rich dynasties.

wealth thresholds, $k^*(a)$, decrease (increase) in e_B if $e_B < e_B^{**}$ ($e_B > e_B^{**}$), where[5]

$$e_B^{**} = \frac{1}{2} > e_B^* \tag{21}$$

As a result, there exists an allocation of public expenditures – noted by $\hat{e}_B$ – such that $\hat{e}_B \in [e_B^*, e_B^{**}]$, below which the number of upwardly mobile individuals is increasing in e_B and above which this number is decreasing in e_B. Figure 5 illustrates the effects of transferring public resources from advanced education to basic education (an increase in e_B) on both fractions of upwardly and downwardly mobile individuals and the resulting stationary proportion of rich dynasties in the long run, $\tilde{R}$. According to the non-monotonic evolution of upward and downward mobility with respect to this transfer, it seems trivial that there exists a certain level of allocation, $\tilde{e}_B$, such that $e_B^* < \tilde{e}_B < \hat{e}_B$, below which the fraction of rich individuals, $\tilde{R}$, increases in e_B, and vice versa.

Proposition 6. Given a fixed size of public education funds, an increase in e_B raises (decreases) the fraction of rich individuals in the long run, $\tilde{R}$, if $e_B < \tilde{e}_B$ ($e_B > \tilde{e}_B$).

[5] For any other values of α and γ, we have $e_B^{**} = \alpha/(\alpha + \gamma)$.

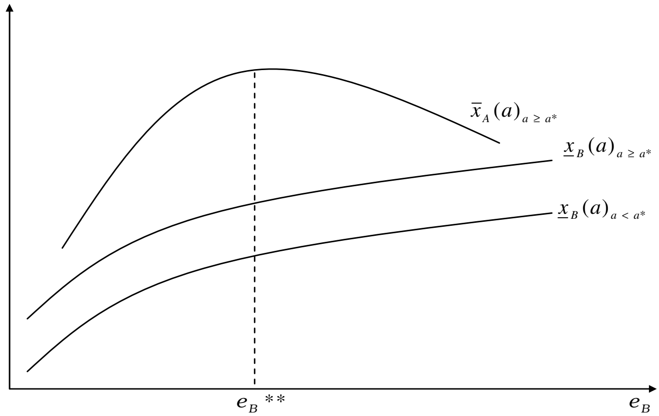

Fig. 6. The effects of expenditure allocation on rich and poor dynasties' long-run levels of wealth.

The effect of this transfer on $\tilde{R}$ reflects the interplay between two conflicting forces: the improvement in the quality of education at the basic level, and the disincentives to acquire education at the advanced level. If the former effect outweighs the latter, upward mobility exceeds downward mobility, so that the equilibrium fraction of rich individuals, $\tilde{R}$, is increasing in this transfer, and vice versa.

How expenditures on education are allocated across basic and advanced stages also has implications on the level of wealth held by each dynasty in the long run.

- According to Equation (12), the lowest long-run levels of wealth, i.e., $\underline{x}_B(a)_{a<a^*}$ and $\underline{x}_B(a)_{a\geq a^*}$, are monotonically increasing in e_B.
- However, Equation (14) shows that the highest long-run levels of wealth, $\bar{x}_A(a)_{a\geq a^*}$, increase (decrease) in e_B as long as $e_B < e_B^{**}$ ($e_B > e_B^{**}$), where e_B^{**} has been defined in Equation (21). We illustrate these relationships in Figure 6.

Proposition 7. Given a fixed size of public education funds, an increase in e_B raises the long-run levels of wealth held by poor dynasties. The increase in e_B also raises the long-run levels of wealth held by rich dynasties as long as $e_B < e_B^{**}$, and vice versa.

VI. CONCLUSION

In this paper, we developed an overlapping-generations model of education investment in which credit markets are imperfect, individuals' abilities are heterogeneous and education is modelled as a two-stage process. We showed that there is a possibility of multiple steady-state equilibria with different levels of investment in advanced education, mobility and average wealth, and the specific equilibrium the economy converges to depends on the initial distribution of wealth. More specifically, the more unequal the economy's initial wealth distribution, the lower is that equilibrium.

In addition, we have found that investment in advanced education, interclass mobility and average wealth are increased along the growth process. Indeed, by increasing public expenditures at all levels of education, the growth process relaxes the liquidity constraints on the poor and enhances the incentives to acquire advanced education for the rich. As a result, net mobility and average wealth are shifted up.

Using our model, we analysed the effects of two educational finance policies: an increase in the total budget of education through an increase in the income tax rate; and a reallocation of public resources across basic and advanced stages of education, while holding fixed the level of the education budget. An important result from this analysis is that the effects of both policies differ a lot. We find that provided that the income tax rate is not too distortional, the increase in the education budget is associated in the long run with positive effects on the levels of investment in advanced education, net mobility and the levels of wealth held by both rich and poor dynasties. However, the effect of reallocating educational resources from basic to advanced education on the incentives to acquire advanced education reflects a tension between two effects of opposite signs: a negative effect on the incomes of the poor, which strengthens their liquidity constraints; and a positive effect on the incomes of the rich, which enhances their incentives to acquire advanced education. In particular, there is an optimal allocation of public resources in favour of advanced education, such that beyond this allocation, additional expenses in favour of this schooling level result, in the long run, in lower economic mobility, a lower fraction of skilled individuals and lower levels of wealth that are held by both rich and poor dynasties.

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APPENDIX

TABLE A
Time-line diagram of individuals' activities

	Unskilled worker	*Skilled worker*
Period 3	Labour revenue: $(1-\tau)h_B(2+r)$ Wealth: $x(1+r)$	Labour revenue $(1-\tau)h_A$ Wealth: $[x(1+r)-\phi](1+i)$ *if borrower* $[x(1+r)-\phi](1+r)$ *if lender*
Period 2	Labour revenue: $(1-\tau)h_B$ Wealth: $x(1+r)$	Advanced human capital: h_A Wealth: $x(1+r)-\phi$
Period 1	Basic human capital: h_B Wealth: x	Basic human capital: h_B Wealth: x

Proof of Proposition 2

The ability threshold level in Equation (9) can be written as follows:

$$a^* = \frac{\phi(1+r)}{(1-\tau)(\tau e_B Y_{t-1})^\alpha\left[\tau^\gamma(1-e_B)^\gamma Y_{t-1}^\gamma-(2+r)\right]}$$

Partial derivation with respect to τ gives

$$\frac{\partial a^*}{\partial \tau} = -E_{Bt}^{\alpha}\tau^{-1}\frac{\phi(1+r)}{(1-\tau)D}\left\{E_{At}^{\gamma}[(\alpha+\gamma)(1-\tau)-\tau]\right.$$
$$\left. - C[\alpha(1-\tau)-\tau]\right\} < 0$$

where $E_{At}^{\gamma} = \tau^{\gamma}(1 - e_B)^{\gamma} Y_{t-1}^{\gamma}$; $E_{Bt}^{\alpha} = (\tau e_B Y_{t-1})^{\alpha} C = 2 + r$; and $D = E_{Bt}^{\alpha}\,(E_{At}^{\gamma} - C)$. Clearly, this derivative is always negative since we have $\gamma(1-\tau) > 0$.

The wealth threshold level given in Equation (8) can be written as follows:

$$x_t^*(a) = \frac{(1+i)\phi - (1-\tau)a(\tau e_B Y_{t-1})^{\alpha}[\tau^{\gamma}(1-e_B)^{\gamma} Y_{t-1}^{\gamma} - (2+r)]}{(1+r)(i-r)}$$

The derivation of this expression with respect to τ yields

$$\frac{\partial x_t^*(a)}{\partial \tau} = \frac{-aE_{Bt}^{\alpha}}{\tau(1+r)(i-r)}\left\{E_{At}^{\gamma}\gamma(1-\tau)(1-\alpha)\right.$$
$$\left. - (2+r)[(1-\tau)\alpha - \tau]\right\} < 0$$

This derivative is always negative since we have $\gamma(1-\tau) > 0$.