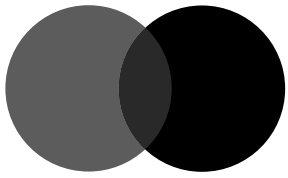


# About the Authors

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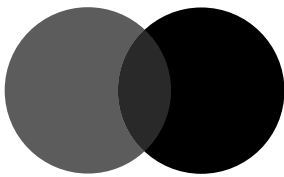
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# Dedications

To my father, George Roberts, who encouraged and nurtured my love and appreciation for mathematics.

To Nancy MacKenzie, and the Cairns sisters, Alexia and Rosemary, my three mothers, without whom none of this would have been possible.




# Preface

This book is intended primarily for use by students and teachers of Higher Level Mathematics in the International Baccalaureate Diploma Programme, but will also be of use to students on other courses.

Detailed coverage is provided for the core part of this syllabus and provides excellent preparation for the final examination. The book provides guidance on areas of the syllabus that may be examined differently depending on whether a graphical calculator is used or not.

Points of theory are presented and explained concisely and are illustrated by worked examples which identify the key skills and techniques. Where appropriate, information and methods are highlighted and margin notes provide further tips and important reminders. These are supported by varied and graded exercises, which consolidate the theory, thus enabling the reader to practise basic skills and challenging exam-style questions. Each chapter concludes with a review exercise that covers all of the skills within the chapter, with a clear distinction between questions where a calculator is

allowed and ones where it is not. The icon  is used to indicate where a calculator

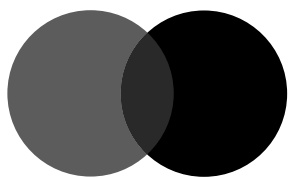
may be used, and  indicates where it may not. Many of these questions are from past IB papers and we would like to thank the International Baccalaureate for permission to reproduce these questions.

Throughout the text, we have aimed to produce chapters that have been sequenced in a logical teaching order with major topics grouped together. However, we have also built in flexibility and in some cases the order in which chapters are used can be changed. For example we have split differential and integral calculus, but these can be taught as one section. Although many students will use this book in a teacher-led environment, it has also been designed to be accessible to students for self-study.

The book is accompanied by a CD. As well as containing an electronic version of the entire book, there is a presumed knowledge chapter covering basic skills, revision exercises of the whole syllabus grouped into six sections, and twenty extended exam-style questions.

We would like extend our thanks to family, friends, colleagues and students who supported and encouraged us throughout the process of writing this book and especially to those closest to each of us, for their patience and understanding.

Bill Roberts  
Sandy MacKenzie  
2007



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