Refractive indices measurement of spherical particles by Fourier Interferometry Imaging

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Abstract In this paper, the measurement of refractive indices of a set of spherical particles is presented. A set of particles is illuminated by a pulsed laser beam. The particles scatter the light toward a CCD camera. A complex interference fringe patterns are recorded by the CCD camera. These interferences fringes depend on these characteristics of the illuminated particles: 3D relative locations, sizes, and refractive indices. This is why it is possible to measure theses parameters from the analysis of interferences fringes. The interference pattern is a complicated 2D signal which may include Moiré effect patterns. In this work we present an analysis of the interference fringes using an approach based on Fourier Transforms, which provide a spectral representation of the fringes. The Fourier space representation of the fringes is a complex matrix characterized by a magnitude spectrum and a phase spectrum. In the Fourier magnitude spectrum, many spots are observed. The spot at the center of the spectrum corresponds to the fringes with low spatial frequency formed by interference between the reflected and refracted light signal scattered by each particle. The spots outside the center of Fourier magnitude spectrum correspond to the fringes with high spatial frequency, corresponding to interferences between light scattered by different particles. For the refractive index measurement of a pair of particles, an individual spot is selected and a rectangular filter centered on the spot is applied. Here, the magnitudes in Fourier space outside the filter are equal to zero. The next step is to take the inverse Fourier transform of the filtered Fourier space and trace the magnitude spectrum. The signal obtained is termed the "scattering composite function". This function depends on refractive indices and sizes of the pair of particles. The inversion for the refractive index measurement is applied to this function.

1. Introduction

The study of disperse two-phase flow requires one to understand and quantify interactions between particles. The aim of the current work is to understand physical phenomena such as evaporation or combustion of a cloud of particles. Many parameters must be measured in order to quantify theses interactions, including the distances between particles, their sizes, velocities, and their representative refractive indices. Refractive index information is heavily influenced by the temperature and the chemical composition of particles.

Many granulometric methods allow one to measure particle information. For example, digital holography can be applied to measure 3D locations and sizes of a set of particles. Global rainbow refractometry can be applied to yield information about refractive indices and sizes of a particle cloud. However, one disadvantage of these methods is that they don't allow the simultaneous measurement of sizes, 3D relative locations and refractive indices (Wu 2012).

By contrast, the Fourier Interferometry Imaging method is well-suited for the simultaneous measurement of 3D relative locations of a set of particles illuminated by a pulsed laser beam (Briard

2011). The FII method was extended to measurement of the sizes of the particles (Briard 2012a). This paper presents the current implementation of the improved FII method.

A numerical simulation code (Wu 2012) was used to validate the conclusions reached in this work concerning the FII method. The code used for validation simulates the light scattering of a set of spherical particles with the help of the near field Lorenz-Mie theory (Slimani 1984). The images presented in this paper are numerical simulation results which come from this code.

In this paper, the second section presents the general principles of FII method. The third section concerns the details of the 2D Fourier Transform. The fourth section presents the refractive index measurement principle. The prospect of applying FII to measure the refractive index is presented in the fifth section.

2. Fourier Interferometry Imaging principle

If many particles are illuminated by a pulsed laser beam, the response of the system can be represented as a set of spherical light waves sources. They create interferences resulting in an interferences fringe field that can be recorded by a CCD camera. The analysis of this signal from the particles permits the measurement of particle characteristics.

In this paper, the CCD camera is located close to the rainbow angle of particles because of the singularity of the scattering function for this angle. The incident wave is a plane wave with wavelength equal to 0.532 µm, and polarization parallel to the axis X. The particles are spherical, homogenous, transparent and isotropic. It is also possible to apply the FII method in other configurations (CCD camera located to another angle, particles inhomogeneous …)

The FII principle is presented in Figure 1.

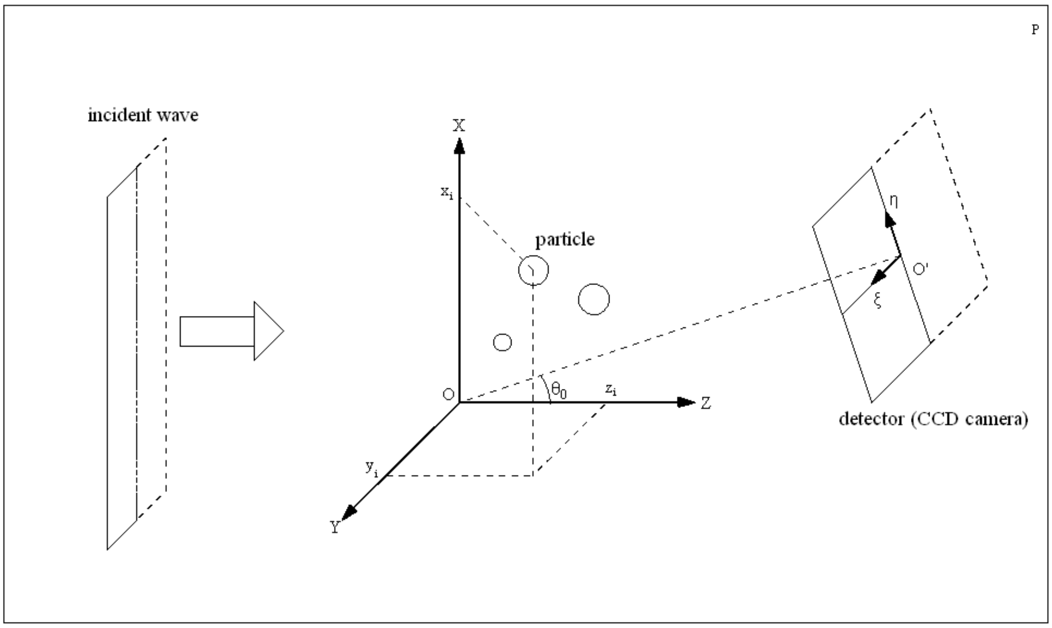


Figure 1. Fourier Interferometry Imaging principle: particles have behavior of spherical waves sources and scatter the light toward a CCD camera.

The centers of particles (figure 1) are expressed in the (OXYZ) system. The location of a point M in the surface of the detector is expressed in the (O’ •⑊ system. The center of the detector O’ is in the

XOZ plan. θ_0 is the angle between Z axis and the OO' line. The complex amplitude of total electric field $E_t(\eta_M, \xi_M)$ in a point M at the surface of the camera CCD takes the form:

$$E_t(\eta_M, \xi_M) = \sum_{k=1}^{N_{part}} E_k(\eta_M, \xi_M) \tag{1}$$

N_{part} is the number of the illuminated particles. k represents a particle and E_k represent the scattered electric field by the particle k.

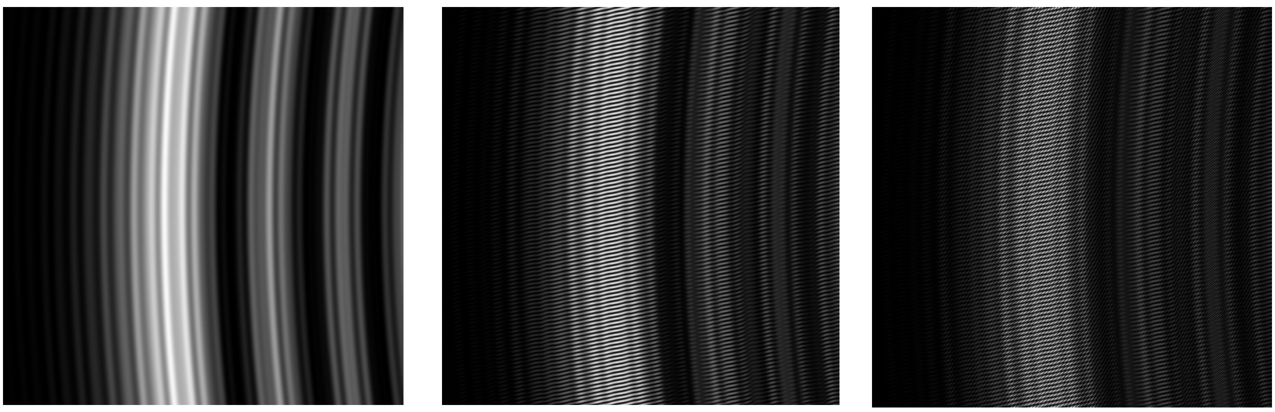
The light intensity $I(\eta_M, \xi_M)$ at point M at the surface of the CCD camera takes the following form:

$$I(\eta_M, \xi_M) = \sum_{k=1}^{N_{part}} E_k^2(\eta_M, \xi_M) + \sum_{k=1}^{N_{part}} \sum_{l=1, l\neq k}^{N_{part}} E_k(\eta_M, \xi_M) E_l^*(\eta_M, \xi_M) \tag{2}$$

E_k^2 corresponds to interferences between refracted and reflected waves scattered by the particle "k". $E_k E_l^*$ is the term corresponding to the interferences between the waves scattered by the particles "k" and "l".

For a single particle, the result is a rainbow (see figure 2.a). The record and the analysis of the rainbow permit the measurement of refractive index with accuracy on the forth decimal because location of rainbow depends on refractive index of the particle.

For more than one particle illuminated, interferences fringes between the waves scattered by particles are recorded by the CCD camera furthermore the rainbows created by the particles. The interferences fringes of the waves scattered by the pair of the particles contain information about refractive indices of the pairs of particles. Examples of global rainbow created by two and three particles are illustrated in figure 2.b and figure 2.c.



a b C

Figure 2. Simulations of interference fringes created by one, two and three particles for CCD camera close to the rainbow angle of the particles (refractive indices are equal to 1.3333, angle θ_0 is equal to 140°).

The interferences fringes system is a very complex 2D signal. The higher the number of particles which are illuminated to generate the signal the higher the complexity of the resulting signal. Moreover, a Moiré effect may be apparent in the signal. For these reasons, the 2D Fourier transform is applied. This is the object of the section 3.

3. 2D Fourier transform of the CCD camera record.

The 2D Fourier transform gives a spectral representation of the interference fringes, with a Fourier magnitude spectrum and a Fourier phase spectrum (figure 3). Afterwards, the interference fringes system and their spectral representations will be referred to as "physical space" and "Fourier space". Fourier space is the complex matrix characterized by phases and magnitudes of the 2D intensity signal.

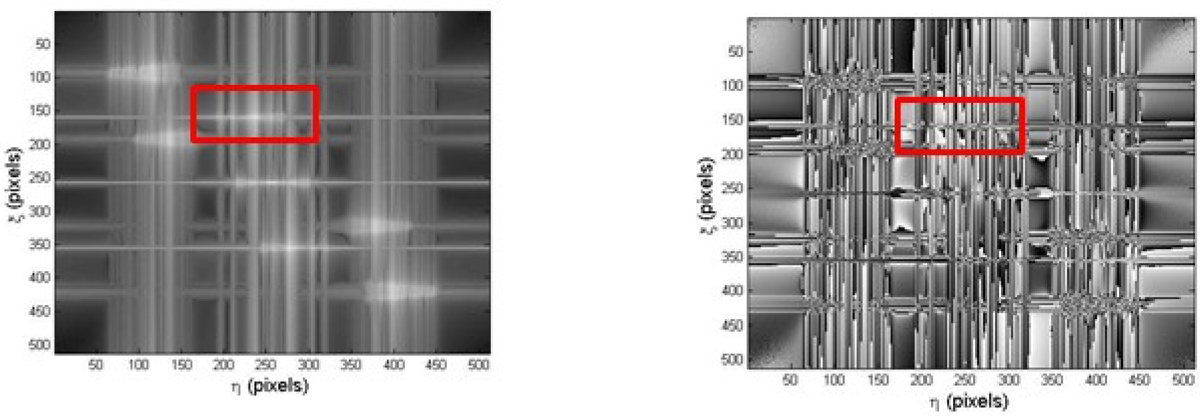


Figure 3. Fourier space: Fourier magnitude spectrum (left) and Fourier phase spectrum (right) of the interferences fringes (figure 2). The red rectangular correspond to a spot created by a pair of particles and the associated phases.

In the Fourier magnitude spectrum (figure 3), many spots are observed. The spot at the center of the magnitude spectrum corresponds to the fringes of interference of the light waves reflected and refracted), scattered by each particles (principally order p=0 and order p=2 for scattering close to rainbow angles of particles). These are the fringes with low spatial frequency associated with the terms $E_k^{\,2}$. The greater the number of illuminated particles, the more complex the analysis required for the central spot.

The spots outside the center of the magnitude spectrum correspond to the interference fringes between the waves scattered by each pair of particles. These are the fringes with high spatial frequency. For a single pair of particles, there are two symmetrical spots beside the central spot in the Fourier Magnitude spectrum, associated to the Fourier transform of the term $E_k E_l^*$.

For the measurement of the refractive indices, one of these spots is selected (inside the red rectangular figure 3). The measurement of refractive indices by the FII method is the object of the forth section.

4. The composite scattering function and the refractive index measurement principle by FII.

We propose in this paper that the inversion can be made in physical space instead of the Fourier space, for each pair of particles. The first step is to apply a rectangular spatial filter in the Fourier space : outside the selected spot, the magnitude is set to zero. The filtered Fourier space is represented in the figure 4.

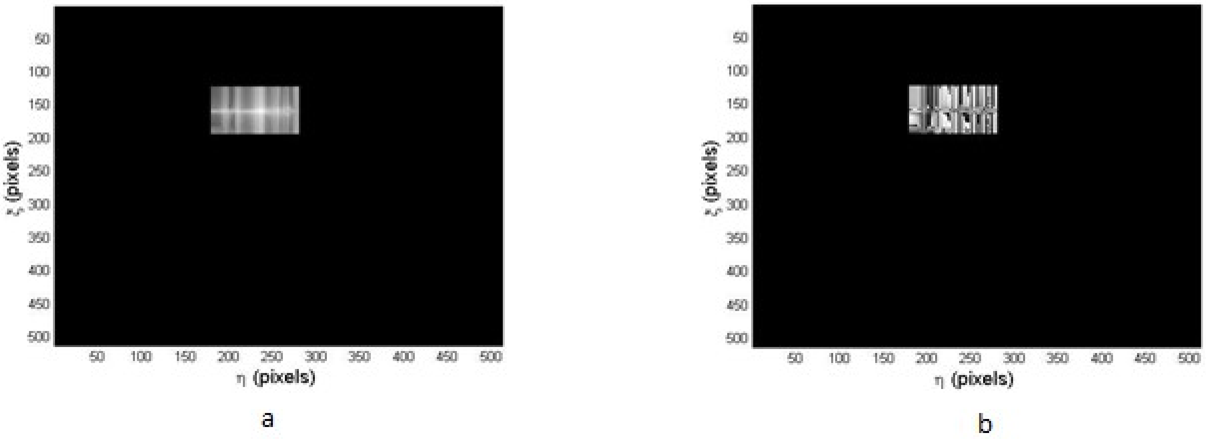


Figure 4. Filtered Fourier space : magnitude Fourier spectrum (a) and phases Fourier spectrum (b). Outside a selected spot in magnitude Fourier spectrum, the magnitude equal to zero.

The next step is to apply the inverse Fourier transform to the filtered Fourier space and trace the magnitude spectrum (figure 5.a). The function obtained is named "2D composite scattering function". The expression of the 2D scattering function $I_{composite}(\eta_M,\xi_M)$ is :

$$I_{composite}(\eta_M,\xi_M) = \sqrt{I_k(\eta_M,\xi_M)\, I_l(\eta_M,\xi_M)} \tag{3}$$

This function depends on diameters and refractive indices of the pair of particles but does not depend on their 3D relative locations. For a CCD camera located close to the rainbow angles of the particles, the function is termed a "2D composite rainbow". The 2D composite rainbow associated with the spot selected in figure 3 is presented figure 5.a. A central profile of the 2D composite rainbow is traced (figure 5.b).

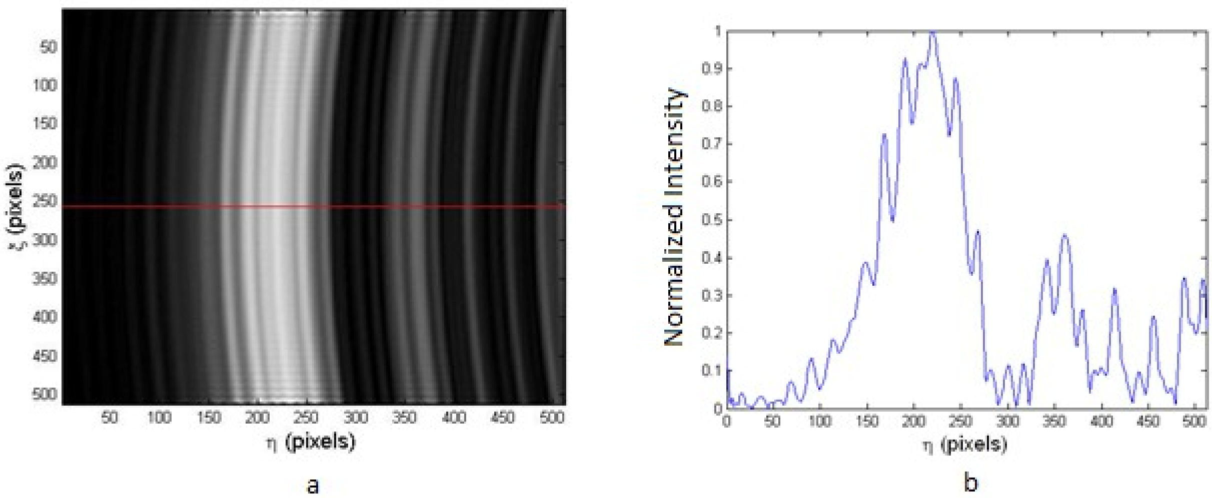


Figure 5. 2D composite rainbow (a) and 1D composite rainbow (b) corresponding by a pair of particles. The particles have refractive index equal to 1.3333 and diameters equal to 100 µm and 130 µm.

We apply the inversion to the 1D composite function in order to measure the refractive index. This is the object of the next section.

4.1 The composite rainbow created by a pair of identical particles

With the equation (3) a particular case can be observed. The figure 6 shows composite rainbow and rainbows created by a pair of identical particles. If the particles are identical, with the same refractive index and the same size, the rainbows created by the particles are identical.

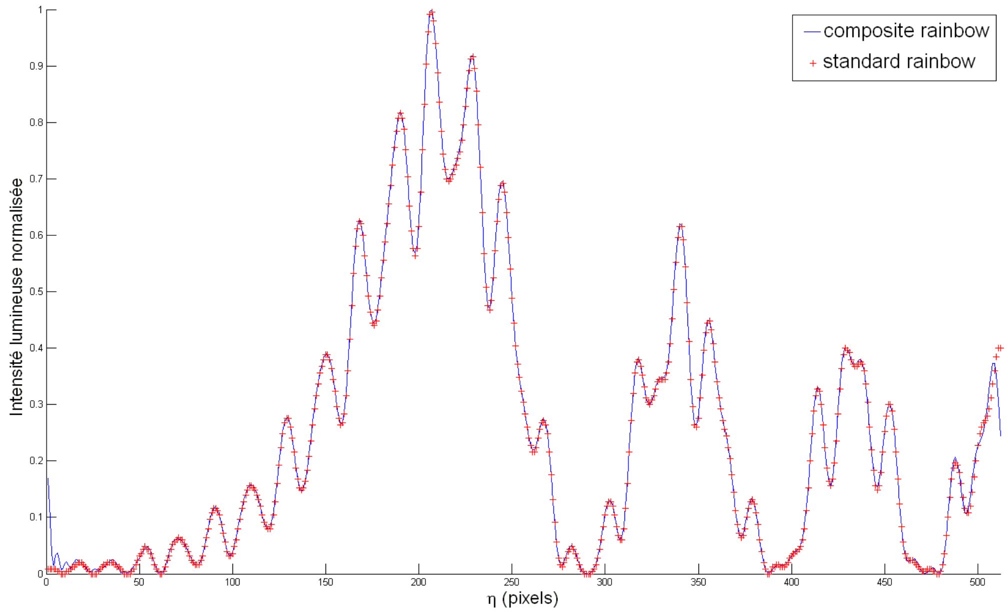


Figure 6. Rainbow created by one particle (blue line) and composite rainbow created by a pair of identical particle with the same characteristics than the one for the individual droplet. Here the particles are water droplets with refractive index equal to 1.3333 and diameter equal to 130 µm.

If the particles of the pair are identical, the inversion of composite rainbow can be made with a standard rainbow inversion code (Saengkaew 2010). Standard rainbow refractometry is refractometry with rainbow created by a single particle. The location of the principal bow depends on the particle characteristics. Applying the standard rainbow inversion code, the refractive index can be measured with an accuracy on to the forth decimal.

4.2 The composite rainbow inversion for different particles

The figure 7 presents composite rainbow and rainbows created by the pair of particles if the particles are different.

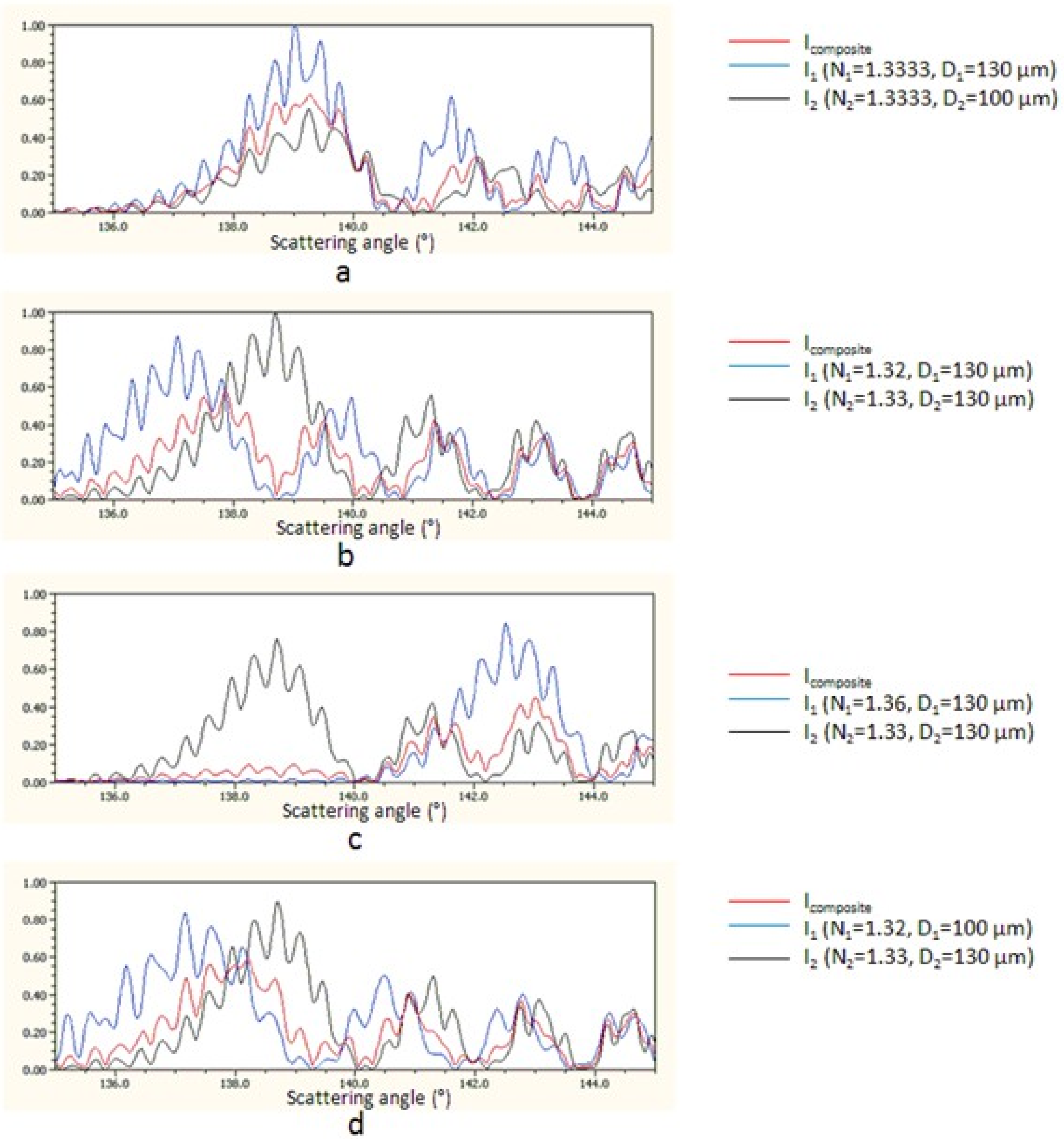


Figure 7. Composite rainbow (red line) created by a pair of particles and the rainbows created by the individual particles in the pair (blue and black lines) if the particles are differents.

In figure 7, there are 3 cases discerned about the pair of different particles:

-Particles have the same refractive index and different size (figure 7.a)

-Particles have different refractive indices and the same size (figure 7.b and 7.c)

-Particles have different refractive indices and different size (figure 7.d)

The composite rainbow obtained by spatial filtering and inverse Fourier transform and the composite rainbow calculated with equation (3) are identical (Briard 2012b). In figure 7, only one curve (composite rainbow calculated from equation (3)) is plotted.

In the figure 7.a where the particles have the same refractive index but different sizes, the composite rainbow is generated at the location as that created by the largest particles. This is why a standard rainbow inversion code is appropriate for measurement of the refractive index of the pair of particles. With an inversion code based on Nussenzweig theory, the measured refractive index from composite rainbow of figure 7.a is equal to 1.33327. The initial refractive indices of the pair of particles were both equal to 1.3333. The potential accuracy for refractive measurement by Fourier interferometry imaging is accuracy on to the forth decimal.

The figures 7.b, 7.c and 7.d show that if refractive indices of the particles of the pair are different, the refractive index measurement with a standard rainbow inversion code isn't possible.

In figure 7.b and 7.d particles have refractive indices with a relative difference; equal to 1.32 and 1.33. The diameters are identical in the case of figure 7.b and different in the case of figure 7.d. The composite rainbow exhibits the shape of the rainbow created by a single particle. The composite rainbow has a main bow located between the principal bow of the rainbows created by the particles. The locations of the three main bows are different. Which is why it is not possible to use standard rainbow inversion code for index measurements if the two particles have different refractive indices.

In the case figure 7.c, refractive indices are equal to 1.33 and 1.36. Here, composite rainbow does not have the shape of a rainbow created by a single particle. Consequently, a standard rainbow inversion code is not useful for refractive indices measurement in this case, even for the measurement of a refractive index between refractive indices of the two particles.

In conclusion, if the particles have the same refractive index, a standard rainbow inversion code can be used to measure it. If the particles don't have the same refractive index, a new strategy for refractive index measurement must be developed. This is the object of the next section.

5. Inversion strategy for a pair of different particles

If the pair of particles have a different refractive index, it is not possible to measure refractive indices with a standard rainbow inversion code.
For development of a new strategy and to see which inversion is possible, it is necessary to study the influence of the refractive indices and diameters on the composite rainbow.
For this, we draw a composite rainbow for different configurations of particles: the refractive indices of the two particles vary between 1.3 and 1.4. The particles have diameter equal to 100 µm.
A reference composite rainbow is chosen for a reference configuration, and it is compared to the ones calculated form the sky composites. The refractive indices chosen for the reference are 1.33 and 1.36.
A quality factor, Q, determines the degree of difference between a composite rainbow created by a particle and the reference composite rainbow:

$$Q = \sum_{\eta=0}^{N_\eta} \left(I_{composite}(\eta) - I_{composite,reference}(\eta) \right)^2 \tag{4}$$

N_η is the number of pixels (512 in this example) in the direction η. The inverse of quality factor Q is calculated as a function of the refractive indices of the pair of particles without noise summed with reference composite rainbow and with noise summed to the reference composite rainbow.

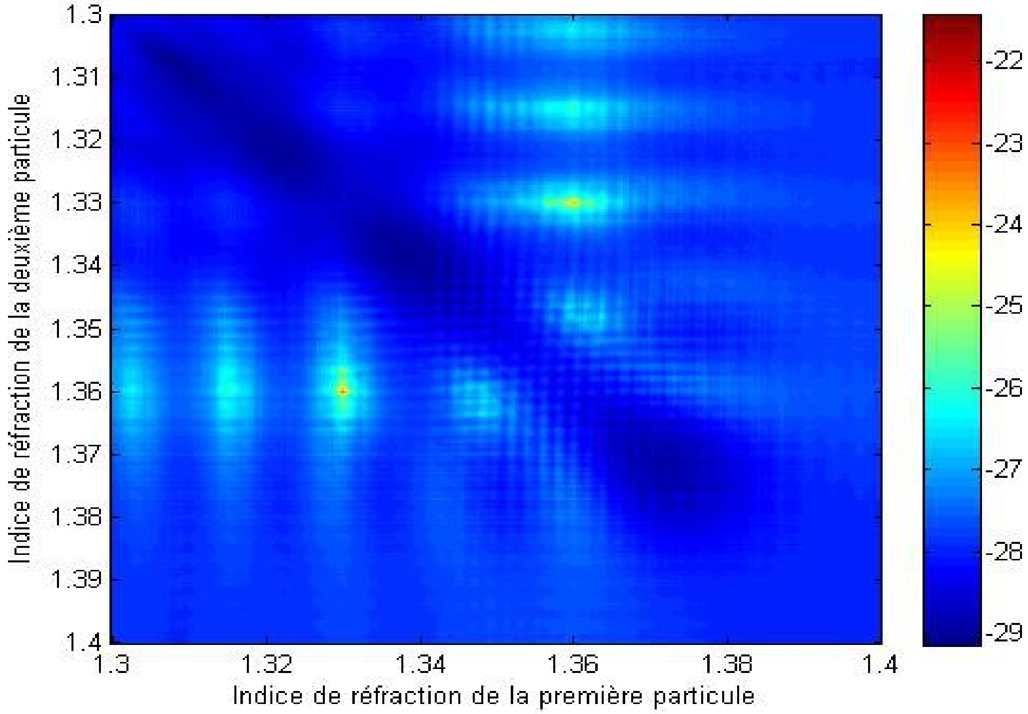


Figure 8. Inverse of quality factor in function to refractive indices of the pair of particles (logarithmic representation).

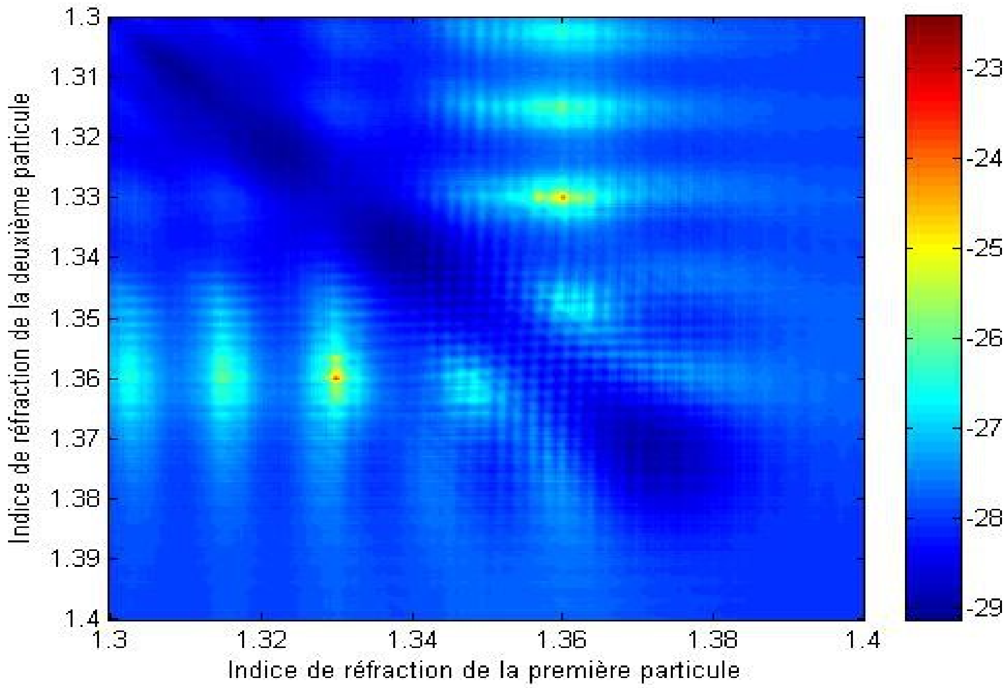


Figure 9. Inverse of quality factor in function to refractive indices of the pair of particles (logarithmic representation). A random noise is summed to the reference composite rainbow with a maximal magnitude equal to 10% of the signal.

The inverse of quality factor decreases if noise is summed to the reference composite rainbow but the map have the same behavior.

On the maps, a set of vertical and horizontal striations and several local maxima which correspond to the main refractive index pairs (1.302, 1.36), (1315, 1.36), and (1.33, 1.36) are observed. The strongest peak intensity corresponds to the pair (1.33, 1.36).

This two-dimensional map shows that the inversion of a rainbow composite to measure the refractive indices of the particles is possible.
However, the optimization algorithm to use for the inversion cannot be a research method of zero gradient, such as Levenberg-Marquardt, because of the many local extrema of the quality factor. An alternative method for reversing the composite rainbow is the application of a genetic algorithm, due to the large number of parameters to optimize (2 diameters and 2 refractive indices) and because it is possible to converge to the correct solution despite the presence of local extrema of the quality factor.

6. Conclusion

The principle of refractive index measurement by FII has been shown in this paper. From spot corresponding to interferences between waves scattered by a pair of particles, a scattering composite function can be constructed. If the CCD camera is located around the rainbow angle of the particle, this function, termed the composite rainbow, depends on refractive indices and sizes of the pair of particles. If the pair of particles has the same refractive index, refractive index can be measured by the application of a standard rainbow code inversion. For the case where particles have different refractive indices, a strategy based on genetically algorithm is in development.

7. Acknowledgments

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