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On an Additional Magnetic Force Present in a System of Coaxial Solenoids

Abstract

In this short article we show by reasoning from the reciprocity relation that some electromagnetic force additional to the Lorentz force should exist in magnetocumulative generators (MCG). The result of action of this force is similar to that of the magnetic pressure, but the origin of this force is quite different.

It is known that the region outside a close-to-ideal (doubly-wound infinite or toroidal) solenoid is effectively 'shielded from the magnetic field,' although the vector potential responsible for the magnetic field is non-zero in that region. So, if we seek to detect effects caused by a hypothetical non-Lorentz force, it will prove advantageous to use solenoid systems for eliminating the background of the Lorentz force as an aid to detecting such (presumably small) additional forces dependent solely on vector potential. One such system is that of two coaxial solenoids (Fig. 1) where, according to our reasoning below, there should logically exist such a non-Lorentz force. That force should be magnetic in nature, since it is determined by magnetic vector potential.

It follows from the reciprocity relation [1]

$$\int\limits_V [\mathbf{B}_1 \times \mathbf{J}_2]\, dv = \int\limits_V [\mathbf{B}_2 \times \mathbf{J}_1]\, dv \qquad (1)$$

where integration is over all space V, that the total force acting on some circuit with current density $\mathbf{J}_1$ from another circuit loaded by the current density $\mathbf{J}_2$ is equal to the force acting from first circuit to second one. In Eq. (1) the circuits are loops of arbitrary shape. Now we apply this theorem to our system of coaxial solenoids (Fig. 1). One can see that this system has the design of a simplest magnetocumulative generator (MCG) [2]; but in our present considerations we limit ourselves to the case of steady-state currents and non-moving coils (so all transient inductive and diffusion processes of the magnetic field through the coils are concluded). To simplify explanation, we call the external circuit a *coil* and the internal circuit a
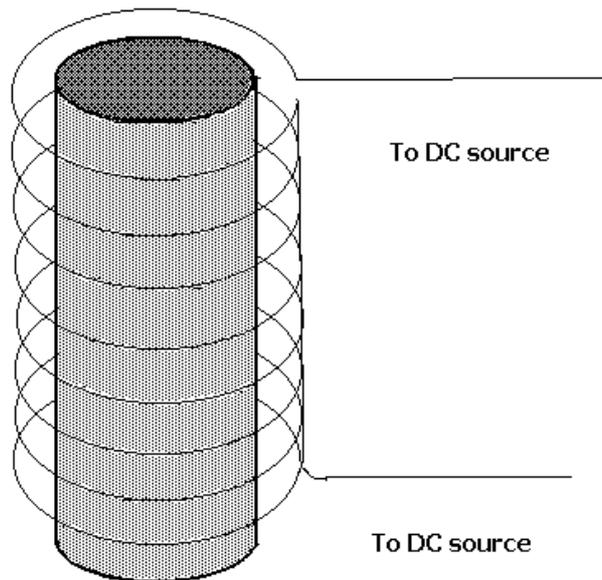

Fig. 1. Sketch of the solenoid (shaded cross section) and coaxial external coil, intended to detect additional force.

solenoid throughout this article. It is only the solenoid that needs to be quasi-ideal.

Now we consider if the reciprocity relation (1) is fulfilled for our system. In the region where the external coil is located, the magnetic field created by the solenoid is equal to zero, so the Lorentz force acting on the coil is equal to zero, too. The Lorentz force acting on each element of the solenoid, however,

is non-zero,

$$\mathbf{F}(c \to s) = -\left[\mathbf{B}_c \times \mathbf{J}_s\right] r_s \delta\phi$$

where $\mathbf{F}(c \to s)$ is the force acting on the element of length $r_s\delta\phi$ of the solenoid from the coil, $\mathbf{B}_c$ is the magnetic field created by the current in the coil and $\mathbf{J}_s$ is the current density in the element of the solenoid. Nevertheless, because of axial symmetry of the solenoid, the *net total* sum of forces acting on all elements of the solenoid is equal to zero — the forces being in dynamic equilibrium. So formally Eq.(1) is fulfilled.

But there is essential difference in effects of action of the force $\mathbf{F}(c \to s)$ and $\mathbf{F}(s \to c)$, the force acting on the element of length $r_c\delta\phi$ of the coil from the solenoid. Because *all* elementary forces $\mathbf{F}(c \to s)$ are directed transversely to the surface of the solenoid, these cause mechanical stress in the solenoid. This implies the possibility of a non-zero radial deformation of the solenoid. Similar mechanical stress is absent in the external coil, if only Lorentz force exists. So, despite the fact that Eq. (1) is satisfied, some difference in force actions on the coil and the solenoid exists and this difference reflects an asymmetry in the effects which the Lorentz force can create when acting between two current circuits. More specifically, in the considered case one Lorentz force produces some work of mechanical deformation (strain) of the circuit it acts on and the second Lorentz force does not produce a corresponding counter-action on the circuit producing the first Lorentz force. Such an apparent contradiction must be resolved and below we explain how this can be done.

It is known that the magnetic Lorentz force is asymmetric and, to show that this asymmetry does not lead to any internal contradictions, it is proven that the Lorentz formula can be reduced to the Ampere formula for the force acting between two current elements (Ch. 5.2 of [3]). For two closed circuits with the currents I_1 and I_2 separated by some distance R_{12} the Ampere force is

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = -\frac{I_1 I_2}{c^2} \oint\limits_{l1} \oint\limits_{l2} \frac{(d\mathbf{l}_1 \cdot d\mathbf{l}_2)\mathbf{R}_{12}}{|\mathbf{R}_{12}|^3} \quad (2)$$

One can easily find that for the closed current circuits, both the Biot-Savart

$$\mathbf{F}_{12,B-S} = -\mathbf{F}_{21,B-S}$$
$$= -\frac{I_1 I_2}{c^2} \oint\limits_{l1} \oint\limits_{l2} \frac{[d\mathbf{l}_1 \times [d\mathbf{l}_2 \times \mathbf{R}_{12}]]}{|\mathbf{R}_{12}|^3} \quad (3)$$

and Ampere (2) formulas give the same result for the forces acting between these circuits. But in addition to the total forces acting between the circuits, one must calculate the torque and possible deformation of the circuits. The magnitudes of torque between two

current circuits are calculated in [4] and it is shown that the torques are antisymmetric,

$$\mathbf{M}_{12,B-S} = -\mathbf{M}_{21,B-S}$$
$$= -\frac{I_1 I_2}{c^2} \oint\limits_{l1} \oint\limits_{l2} \frac{[\mathbf{l}_1 \times [d\mathbf{l}_1 \times [d\mathbf{l}_2 \times \mathbf{R}_{12}]]]}{|\mathbf{R}_{12}|^3}$$

so there is no 'action-counteraction' controversy in respect to force or torque.

But one more quantity, which determines the observable properties of the system, is not yet calculated. This quantity is the magnitude of deformation of the circuit under the force acting transversely to the local axis of the wire. Assuming the elastic modulus or rigidity k of the circuit (in the transverse direction) is approximately constant along the length of the circuit, we can calculate observable strain as a function of the absolute value of transverse force acting on the wire. Because the Lorentz force acting on a unit element $d\mathbf{l}_1$ of the first circuit from the second circuit is

$$d\mathbf{F}_{21,B-S} = -\frac{I_1 I_2}{c^2} \oint\limits_{l2} \frac{[d\mathbf{l}_1 \times [d\mathbf{l}_2 \times \mathbf{R}_{12}]]}{|\mathbf{R}_{12}|^3}$$

where $\mathbf{n}_1$ is the unit vector in $d\mathbf{l}_1$ direction, so the total deformation Def_{21} of the first circuit caused by the Lorentz force is

$$Def_{21} = \frac{I_1 I_2}{kc^2} \oint\limits_{l1} dl_1 \left| \oint\limits_{l2} \frac{[\mathbf{n}_1 \times [d\mathbf{l}_2 \times \mathbf{R}_{12}]]}{|\mathbf{R}_{12}|^3} \right| \quad (4)$$

Unfortunately, we cannot reduce Eq.(4) to a simpler expression, but we note that Eq.(4) correctly describes the value of deformation, at least for the coil-solenoid system under consideration. One can see that the total value of deformation of the external coil is equal to zero (if we use the Biot-Savart formula (3) for the force) and the value of deformation of the solenoid is non-zero.

This is not the case if we use the Ampere formula for the force, because the Ampere expression

$$|\delta\mathbf{F}_{12}| = -|\delta\mathbf{F}_{21}| = -\frac{I_1 I_2}{c^2} \left| \frac{(d\mathbf{l}_1 \cdot d\mathbf{l}_2)\mathbf{R}_{12}}{|\mathbf{R}_{12}|^3} \right|$$

is originally *symmetric* so the total deformations of both circuits must be the same for the Ampere force[1]. Obviously, within the frame of the Maxwell-Lorentz electrodynamics, it is impossible to save the balance of energy. One can suggest a possible resolution of this paradox as follows:

The local[2] reciprocity theorem should be formulated not for the magnetic field and the current

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[1] Ampere's original formula, from which the above derives, being determined by the imposition of Newton's third law for actions between individual current elements.

[2] Meaning the form written for two interacting elements of the circuit but not for the circuit as a whole.

creating the expression for the *force* but for the current and the vector potential, *i.e.*, the quantities creating the expression for the electromagnetic *energy*:

$$\int_{V_1} (\mathbf{A}_1 \cdot \mathbf{J}_2)\, dv = \int_{V_2} (\mathbf{A}_2 \cdot \mathbf{J}_1)\, dv \qquad (5)$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the vector potentials created by the first and second currents, respectively, and integration is performed over the volumes V_1 and V_2 of the current elements. Now we derive the expression for the electromagnetic force based on the interaction energy in the form (5). We use for this a Lagrangian approach (Sec. 17 of [5]).

Because the most general expression for the electromagnetic force cannot depend on the system for which this expression is applicable, in order to derive the formula for the electromagnetic force we may consider the simplest system, say, a charged particle p moving in some region with given vector potential $\mathbf{A}_s$ exterior to the solenoid. Then, according to our suggestion (5), the energy interaction term $\mathcal{H}_{int}$ should have the form

$$\mathcal{H}_{int} = -\frac{1}{c}\int \left(\mathbf{j}(\mathbf{r}-\mathbf{r}_p)\cdot \mathbf{A}_s(\mathbf{r})\right) dV$$

So the Lagrange equation for p takes the form

$$\frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}_p} = \frac{\partial L}{\partial \mathbf{r}_p}$$

where the Lagrangian L is determined by analogy with Eq. (16.4) of [5] as

$$L = \frac{1}{2}m\mathbf{v}_p^2 + \frac{1}{c}\int \left(\mathbf{j}(\mathbf{r}-\mathbf{r}_p)\cdot \mathbf{A}_s(\mathbf{r})\right) dV$$

We omit the term containing the magnetic fields in the above expression because the coordinate of the particle p does not enter it explicitly.

Thus, the Lagrange equations take a form

$$\frac{d}{dt}\left(m\mathbf{v}_p + \frac{q}{c}\mathbf{A}_s(\mathbf{r}_p)\right)$$
$$= \frac{1}{c}\int \nabla_r \left(\mathbf{j}(\mathbf{r}-\mathbf{r}_p)\cdot \mathbf{A}_s(\mathbf{r})\right) dV \quad (6)$$

It should be noted that if in the *lhs* of Eq.(6), the velocity $\mathbf{v}_p$ is considered as a velocity of the elementary charge q as a whole (we treat the charge as non-deformed when it moves), the strict procedure of solving prohibits use of the point charge approximation in the *rhs* of Eq.(6), if only to avoid divergences in self-energy terms appearing when the radius of the charge tends to zero. Further, for the model of extended nonrelativistic charge (for example, the model of Abraham-Lorentz, Ch. 16.3 of [3]), the external force (in our case, the *rhs* of Eq.(6)) should

be calculated as a sum of forces created by each elementary volume of the charge, so the gradient should be calculated before integration in Eq.(6) over the whole space. This can be strictly shown by introducing the velocity distribution for the charge.

Applying a well-known vector identity (Ch. 5.5-2. of [6]), we obtain from (6)

$$\frac{d}{dt}\left(m\mathbf{v}_p + \frac{q}{c}\mathbf{A}_s(\mathbf{r}_p)\right)$$
$$= \frac{1}{c}\int \left(\mathbf{j}(\mathbf{r}-\mathbf{r}_p)\cdot \nabla_r\right)\mathbf{A}_s(\mathbf{r})dV$$
$$+ \frac{1}{c}\int \left(\mathbf{A}_s(\mathbf{r})\cdot \nabla_r\right)\mathbf{j}(\mathbf{r}-\mathbf{r}_p)dV$$
$$+ \frac{1}{c}\int \left[\mathbf{j}(\mathbf{r}-\mathbf{r}_p)\times \left[\nabla_r \times \mathbf{A}_s(\mathbf{r})\right]\right] dV$$
$$+ \frac{1}{c}\int \left[\mathbf{A}_s(\mathbf{r})\times \left[\nabla_r \times \mathbf{j}(\mathbf{r}-\mathbf{r}_p)\right]\right] dV. \quad (7)$$

We note that the operation

$$\left(\mathbf{A}_s(\mathbf{r})\cdot \nabla_r\right)\mathbf{j}(\mathbf{r}-\mathbf{r}_p)$$

is correctly defined in the classical electrodynamics even for the limit of the point charge because the current density is a vector field existing throughout space ($\mathbf{j}(\mathbf{r}-\mathbf{r}_p)$ is actually some confined function, *i.e.* local; however, it is defined in the whole space). We point out that a similar operation of action of the differential operator ∇ on $\mathbf{j}(\mathbf{r}-\mathbf{r}_p)$ is used in the law of conservation of charge.

Now we return to Eq.(7). The Lorentz force is the third term on the *rhs* here. Because the total time derivative of vector potential consists of two terms, *i.e.*, of the term describing the change of vector potential in time at a given point of space and of the term describing the change of vector potential due to charge motion between neighboring points of space, we have

$$\frac{d\mathbf{A}_s}{dt} = \frac{\partial \mathbf{A}_s}{\partial t} + (\mathbf{v}_p\cdot \nabla)\mathbf{A}_s \qquad (8)$$

But the second term on the *rhs* of Eq.(8) is equal to the first term on the *rhs* of Eq.(7) (the former being actually the latter calculated in the point-charge limit). Also under our assumed condition, $\partial A_s/\partial t = 0$ and $[\nabla_r \times \mathbf{A}_s(\mathbf{r})] = 0$ (outside an ideal solenoid), therefore, Eq.(7) reduces to

$$\frac{d(m\mathbf{v}_p)}{dt} = \frac{1}{c}\int \left(\mathbf{A}_s(\mathbf{r})\cdot \nabla_r\right)\mathbf{j}(\mathbf{r}-\mathbf{r}_p)dV$$
$$+ \frac{1}{c}\int \left[\mathbf{A}_s(\mathbf{r})\times \left[\nabla_r \times \mathbf{j}(\mathbf{r}-\mathbf{r}_p)\right]\right] dV. \quad (9)$$

Because the *lhs* of Eq.(9) is the total time derivative of the momentum of the charge, the *rhs* of this equation describes the total force acting on the charge passing near the solenoid.

Consider p to be a conduction electron in the coil of Fig. 1. The first term on the *rhs* of Eq.(9) is then

a directional derivative along the wires of the coil, and therefore is not relevant to the present (radial force) considerations. However, the direction of the vector corresponding to the second term on the *rhs* of Eq.(9) is opposite to the direction of the Lorentz force vector. This radial force term is in general non-zero because of curvature (non-zero curl of **j**) of the coil wire. We may conclude that the only force (equal and oppositely directed) available to compensate action of the Lorentz force on the solenoid, is the (radial) force acting on the coil, described by the last term of (9).

Finally, we note that the physical effect created by this extra force in the MCG is similar to the effect of the magnetic pressure on the external coil driven by the explosive matter. However, there is an essential difference between these effects, since this new force is able to act on a non-moving external coil, too. From the authors' point of view, one ought to take into account the influence of this force in accurate calculations of work of the MCGs. More generally, experiments to validate or disconfirm, in suitable geometries, the various terms of our proposed generalized equation of motion (7) would evidently be desirable. The Lorentz force is the only one of those terms currently known beyond doubt to be physically valid. For a century too much has been left to accepted electromagnetic theory and not enough to empirical inquiry.

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