

# Robust Sensorless sliding mode Control with Luenberger Observer design applied to Permanent Magnet Synchronous Motor

I. Bakhti, S. Chaouch, A. Makouf and T. Douadi

**Abstract:** This paper addresses the robust stabilization problem of a permanent magnet synchronous motor (PMSM). In order to optimize the speed-control performance of the PMSM system with different disturbances and uncertainties, Sliding mode control design for the PMSM is developed. We discuss in this paper how to achieve and maintain the prospective benefits of sliding mode control (SMC) methodology combined with the Luenberger observer design for on-line estimation of speed and position. The proposed sensorless nonlinear control is theoretically analyzed and assessed in simulation with satisfactory results.

**Keywords:** Sensorless control, Sliding Mode Control, Permanent Magnet Synchronous Motor, Luenberger observer.

## I. INTRODUCTION

In our fast-paced world, permanent magnet synchronous motors commonly used in industrial automation for traction, robotics or aerospace require greater power and heightened intelligence. The efficiency of electrical machine drives is greatly reduced at light loads, where the flux magnitude reference is held on its initial value. Moreover, expert control algorithms are employed in order to improve machine performance [1-3].

One of the important and the famous controls for non linear systems is Sliding Mode (SMC). Due to its robustness against a large class of perturbations or model uncertainties, the need for a reduced amount of information in comparison to classical control and also the possibility of stabilizing some non linear systems which are not stabilizable by continuous state feedback laws make SMC the more attractive controls in the last recent years [4-8].

Moreover, another interesting peculiarity of the sliding mode behaviour is that, because of the geometrical constraint represented by the sliding mode design, a system in sliding mode behaves as a system of reduced order respect the original plant.

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In order to evaluate the SMC, an observer design called Luenberger is presented in the next section.

Design of observers is usually considered as a graduate level topic and taught in a graduate level control engineering course. However, in the most recent editions of several standard undergraduate control system textbooks we can find the coverage of full-order and even reduced-order observers [9].

A state observer based on sensorless control strategy is a good solution for a wide range of fixed speed and low cost applications such as fuel pumps or fans. In a state observer the complete differential motor model is used to estimate the whole state variable which includes both the (unknown) rotor speed and position and the (measurable) motor currents. The observer needs relative accuracy in the modeling of the equation of the unknown variables, the measurements of the motor currents, and the knowledge of the feeding voltages [10-11].

The suggested control scheme, as a result, achieves a sound performance with computational complexity reduction on obtained by using the analytical relation to determine the Luenberger Observer gain matrix. The observer is simple and robust, when compared with the previously developed observers, and suitable for online implementation [12-13]. In this work the Luenberger state observer design is used in order to estimate speed and position.

This paper is organized as follows; the mathematical model of PMSM is described in section 2, Sliding Mode Control Design is presented in section 3 and the Luenberger observer design in section 4; deals with the simulation results. Finally some concluding remarks end the paper.

## II. MATHEMATICAL MODEL OF THE PMSM

The model of a typical PMSM can be described in the well-known (d-q) frame through the Park Transformation as follows:

$$\begin{bmatrix} \dot{I}_d \\ \dot{I}_q \\ \dot{\Omega} \end{bmatrix} = F + G U(1)$$

With

$$U = [V_d \quad V_q]^T$$

$$F = \begin{bmatrix} -\frac{R_s}{L_d}I_d + \frac{L_q}{L_d}p\Omega I_q \\ -\frac{R_s}{L_q}I_q - \frac{L_d}{L_q}p\Omega I_d - \frac{\phi_f}{L_q}p\Omega \\ \frac{3p}{2J}[(L_d - L_q)I_d I_q + \phi_f I_q] - \frac{f}{J}\Omega - \frac{T_l}{J} \end{bmatrix}$$

And

$$G = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_d} \\ 0 & 0 \end{bmatrix}$$

Where:

(*d, q*) Axes for direct and quadrature park subscripts.

$R_s$  Stator resistance.

$L_d, L_q$  Self inductance indirect and quadrature park subscripts

$J$  Inertia moment of the moving element

$f$  Viscous friction and iron-loss coefficient.

$V_d, V_q$  Stator voltage in direct and quadrature park subscripts

$I_s$  Stator Currents

$T_l$  Load torque.

$p$  Is number of pole pairs

$\phi_f$  flux.

$\Omega$  Rotor speed.

### III. SLIDING MODE CONTROL DESIGN

The sliding mode control can be justified and designed using the notion of Lyapunov stability. By solving the equation  $\dot{S} = 0$ , the equivalent control  $U_{eq}$  can be obtained. The  $U_n$  component satisfies  $S\dot{S} < 0$  and is given by:

$$U_n = -k \text{sign}S \quad (2)$$

With:  $k > 0$

#### A. Selection of Switching Surfaces and Determination of the Control Inputs

We use attractivity condition of switched surface  $S\dot{S} < 0$ . The vector of control laws can be expressed as:

$$U = U_{eq} + U_n \quad (3)$$

Surfaces are chosen in order to determine the behavior of the motor in the transient period. For the speed control, we propose switching law which depends on the difference between reference speed and real speed, presented in (4):

$$S(\Omega) = (\Omega_{ref} - \Omega) \quad (4)$$

With  $\Omega_{ref}$  is the rotor speed reference.

The derivative of this surface is given by the expression:

$$\dot{S}(\Omega) = \dot{\Omega}_{ref} - c_1\Omega + \frac{T_l}{J} - (c_2I_d + c_3)I_q \quad (5)$$

With:

$$c_1 = -\frac{f_r}{J}, c_2 = \frac{p(L_d - L_q)}{J}, c_3 = \frac{p\phi_f}{J}$$

The associated control input is given by (6):

$$I_{qref} = \frac{-c_1\Omega + \frac{T_l}{J} + \dot{\Omega}_{ref} + k_{\Omega} \text{sign}S(\Omega)}{c_2I_d + c_3} \quad (6)$$

With the speed gain  $k_{\Omega} > 0$

The components  $I_d$  and  $I_q$  are independently controlled.

$$S(I_d) = (I_{dref} - I_d)S(I_q) = (I_{qref} - I_q) \quad (7)$$

With  $I_{dref} = 0$

Frequently  $I_{dref}$  is made equal to zero, because its contribution to the motor torque is almost insignificant. Flux and torque control are independently made through the surfaces  $S(I_d)$  and  $S(I_q)$  respectively.

The derivative of the surface  $S(I_d)$  and  $S(I_q)$  is given by the expression:

$$\dot{S}(I_d) = \dot{I}_{dref} - a_1I_d - a_2I_q\Omega + \frac{1}{L_d}V_d \quad (8)$$

$$\dot{S}(I_q) = \dot{I}_{qref} - b_1I_q - b_2I_d\Omega - b_3\Omega + \frac{1}{L_q}V_q$$

With:

$$a_1 = -\frac{R_r}{L_d}, a_2 = \frac{pL_q}{L_d}, b_1 = -\frac{R_r}{L_q}, b_2 = -\frac{pL_d}{L_q}$$

$$b_3 = -\frac{p\phi_f}{L_q}$$

The associated control inputs is given by (9):

$$U_{dref} = \frac{[\dot{I}_{dref} - a_1I_d - a_2I_q\Omega] + k_d \text{sign}S(I_d)}{L_d} \quad (9)$$

$$U_{qref} = \frac{[\dot{I}_{qref} - b_1I_q - b_2I_d\Omega + b_3\Omega] + k_q \text{sign}S(I_q)}{L_q}$$

Hence  $k_d$ ,  $k_q$  and  $k_{\Omega}$  are positives gains, given as followed:

$$k_d = 3000, k_q = 4000, k_{\Omega} = 1$$

The necessity for high performance in PMSM systems increases as the demand for precision control is necessary to estimate the rotor speed and the position. For this, the Luenberger observer design is presented in the next section.

#### IV. LUENBERGER OBSERVER DESIGN

The theory of observers originated in the work of Luenberger in the middle of the 1960[10]-[12]. According to Luenberger, any system driven by the output of the given system can serve as an observer for that system. Consider a linear dynamic system :

$$\begin{aligned} \dot{x} &= A(x) + Bu \\ y &= C(x) \end{aligned} \quad (10)$$

Where  $x$  is the state space vector of dimension  $n$ ,  $u$  is the system input vector (that may be used as a system control input) of dimension  $m$ , and matrices  $A$  and  $B$  are constant and of appropriate dimensions. In general, the dimension of the output signal is much smaller than the dimension of the state space variables, that is,  $\dim\{y(t)\} = l = c < n = \dim\{x(t)\}$ , and  $c = \text{rank}\{C\}$ .

In our work we present  $x$  as follow:

$$x = [\theta_r, \omega_r, T_l]$$

Where  $\theta_r$  is rotor position and  $\omega_r$  is the rotor angular frequency

$$\text{And: } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{f}{J} - \frac{1}{J} & 0 \\ 0 & 0 & 0 \end{bmatrix}; B = [0 \quad \frac{p\phi_f}{J} \quad 0]^T$$

$$C = [1 \quad 0 \quad 0]$$

Hence, we have also assumed that there are not redundant measurements. In such a case, under certain conditions, we can use an observer, a dynamic system driven by the system input and output signals with the goal to reconstructing (observing, estimating) at all times all the system state space variables as presented in figure (1).

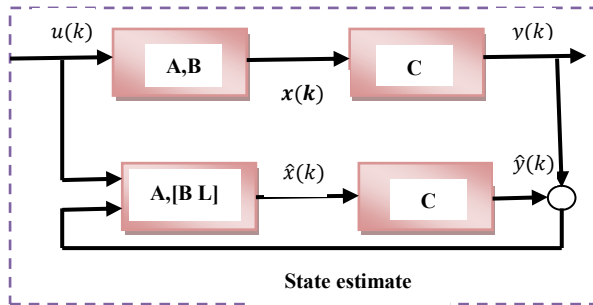


Figure. 1 Luenberger observer design

As constructed in the previous section, an observer has the same structure as the original system plus the driving feedback term that carries information about the observation error. The state model of Luenberger observer is given by the follow equation,

Since the matrices  $A, B, C$  are known, it is rational to postulate an observer as (11):

$$\begin{aligned} \frac{d}{dt} \hat{x} &= (A - LC)\hat{x} + Bu + L y(x) \\ y &= C(x) \end{aligned} \quad (11)$$

To ensure that the estimation error vanishes over time for any  $\hat{x}(0)$ , we should select the observer gain matrix  $L$ . So that  $(A - LC)$  is asymptotically stable. Consequently, the observer gain matrix should be chosen so that all eigenvalues of  $(A - LC)$  have real negative parts. For all these conditions the matrix gain  $L$  is represented as follow:

$$L = \begin{bmatrix} 0 & 0 & 0 \\ l_2 & l_1 & 0 \\ l_3 & 0 & 0 \end{bmatrix}$$

With:  $l_1, l_2, l_3$  are positives gains

The error dynamic of observer is given by equations (12) as follow, then the estimation error  $e(t)$  will decay to zero for any initial condition:

$$\dot{e}(t) = (A - LC)e(t) \quad (12)$$

With:  $e(t) = x - \hat{x}$

The state Luenberger observer equations can be written by the following equations:

$$\begin{aligned} \frac{d\hat{\theta}_r}{dt} &= \hat{\omega}_r \\ \frac{d\hat{\omega}_r}{dt} &= \frac{1}{J} (c_{em} - \hat{T}_l) - \frac{f}{J} \hat{\omega}_r + l_1(\omega_r - \hat{\omega}_r) + l_2(\theta_r - \hat{\theta}_r) \\ \frac{d\hat{T}_l}{dt} &= l_3(\theta_r - \hat{\theta}_r) \end{aligned} \quad (13)$$

Figure(2) present a structure of Sliding mode control combining with Luenberger observer

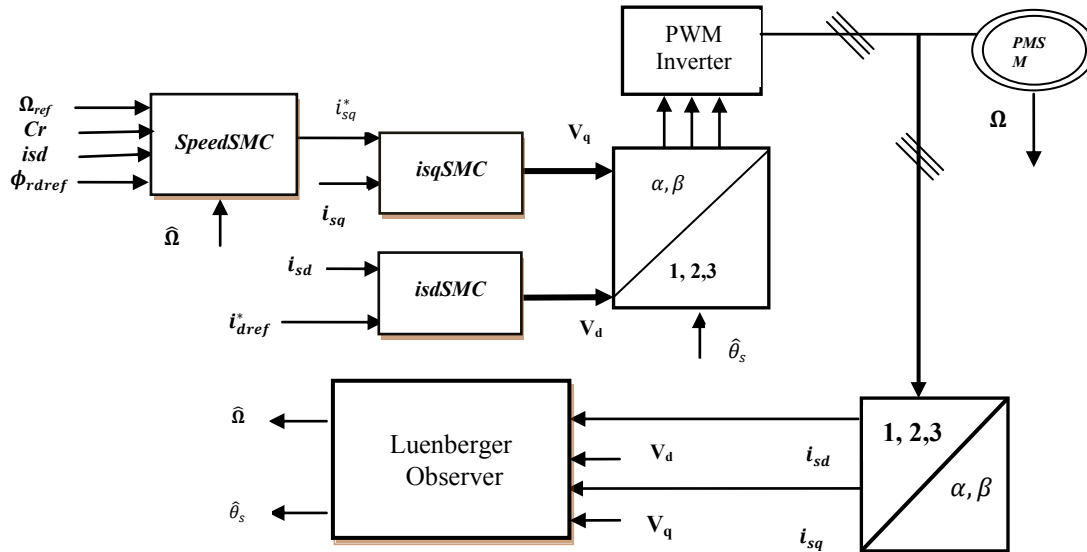


Figure. 2 Configuration of the Sliding Mode control with Luenberger observer

## V. SIMULATION RESULTS

The parameters of the used motor are given in the table (1). The performance of the motor when a load torque applied to the machine's shaft is originally set to its nominal value (0N.m) and step up to 10 N.m at  $t = 0.2$  s, and the desired speed is 200rad/sec. We can mention good results at time of load torque variation for speed, position and load torque proved by speed error turn around zero under a short time and high load torque. Luenberger observer presents a fast and smooth dynamic response for PMSM speed control. In order to evaluate sensorless non linear controls combined with Luenberger properties, we will realize a robustness test.

Table 1 Motor Parameters

$R_s$	0.12 $\Omega$	$L_d$	0.0014H
p	4	$L_q$	0.0028H
$f$	0.0014	$J$	0.0011kgm <sup>2</sup>

The speed tracking controller is operated in a critical situation (rapidly changes as 200,-200, 5 rad/s), and it can be noticed that the proposed observer works in very low speed region, we affect also changes in load torque of 5 to -5 N.m according at time  $t=0.1$  and 0.3s and a variation of 100% of the nominal stator resistance between time  $t = 0.1$ s

and  $t=0.3$ s. Figure (3) shows the satisfactory performances of the speed tracking and position with his estimate. We can see that the actual speed follows the speed command and estimated speed and stator resistance variation is negligible. Thus, the simulation results confirm that the proposed observer gives good results justified by rotor speed error and position error converges to zero rapidly.

## VI. CONCLUSION

In this paper, for a sensorless speed response, a sliding mode control (SMC) combined with Luenberger observer is proposed. In order to offer a good choice of design tools to accommodate uncertainties and nonlinearities, the dynamics behaviour and the control performances obtained are satisfactory, the perturbation is rejected. This study has demonstrated that the design using sliding mode control is successful and able to exhibit excellent robustness due to uncertainties in the Sensorless speed based Luenberger observer design model.

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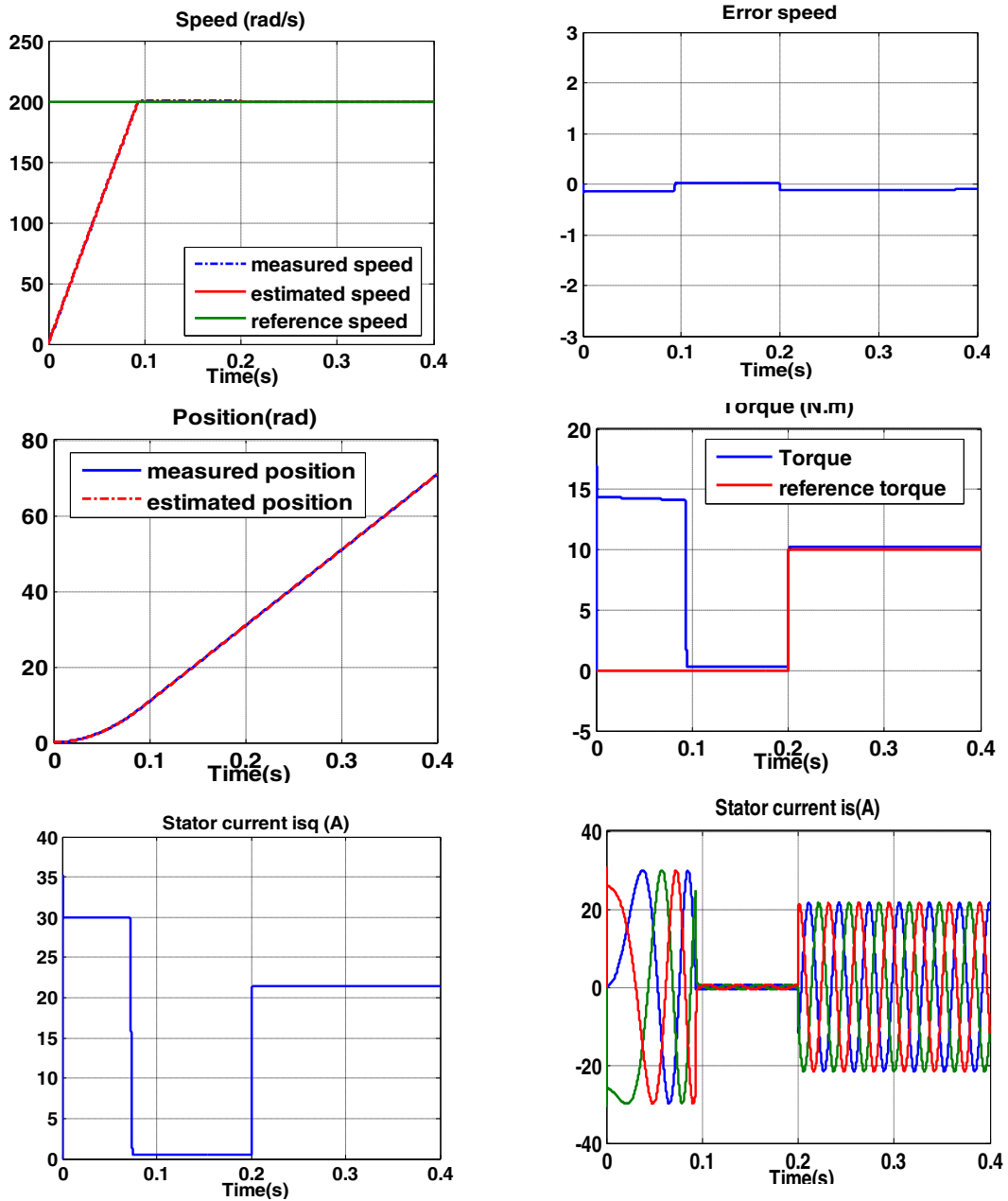


Figure 3. Simulation results with load torque variation:

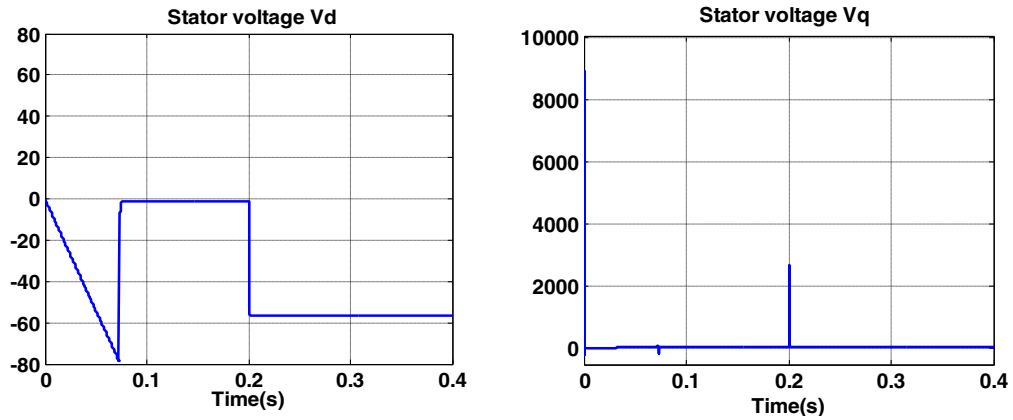


Figure 3. Simulation results with load torque variation of stator voltages

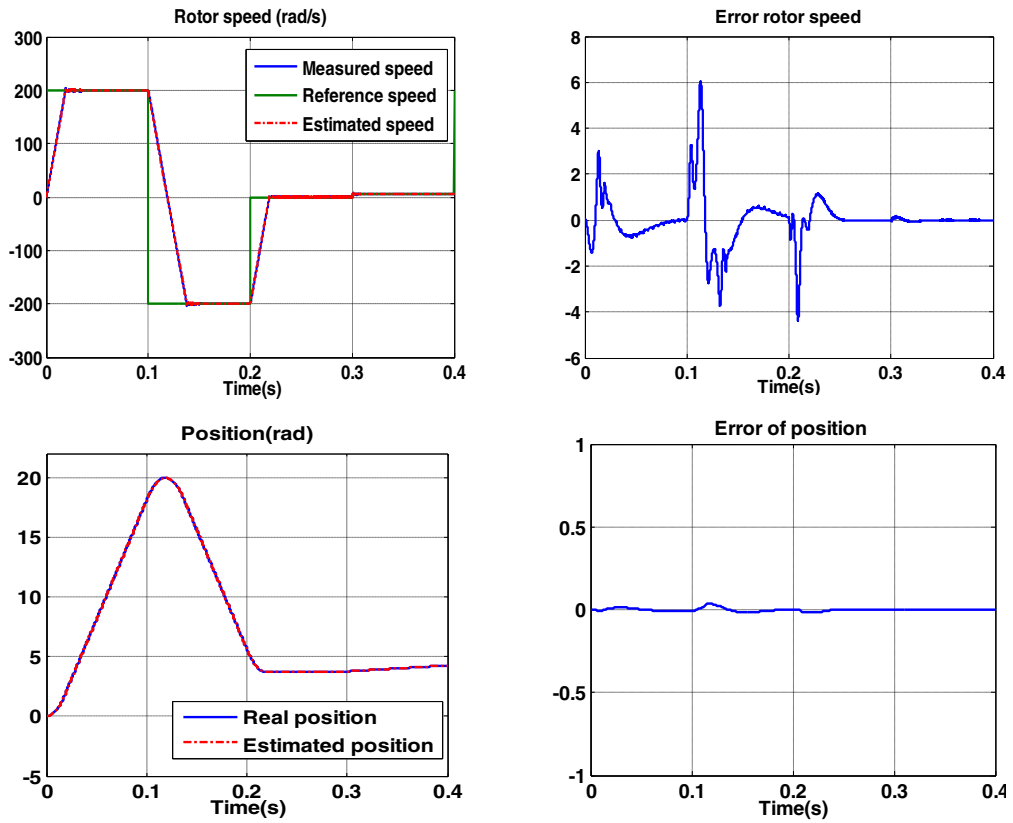


Figure.4 Simulation results with rotor speed variation