SAP2000®

CONCRETE SHELL REINFORCEMENT DESIGN

Technical Note

Design Information

Background

The design of reinforcement for concrete shells in accordance with a predetermined field of moments, as implemented in SAP2000, is based on the following two papers:

- "Optimum Design of Reinforced Concrete Shells and Slabs" by Troels Brondum-Nielsen, Technical University of Denmark, Report NR.R 1974
- "Design of Concrete Slabs for Transverse Shear," Peter Marti, *ACI Structural Journal*, March-April 1990

Generally, slab elements are subjected to eight stress resultants. In SAP2000 terminology, those resultants are the three membrane force components f_{11}, f_{22} and f_{12}; the two flexural moment components m_{11} and m_{22} and the twisting moment m_{12}; and the two transverse shear force components V_{13} and V_{23}. For the purpose of design, the slab is conceived as comprising two outer layers centered on the mid-planes of the outer reinforcement layers and an uncracked core—this is sometimes called a "sandwich model." The covers of the sandwich model (i.e., the outer layers) are assumed to carry moments and membrane forces, while the transverse shear forces are assigned to the core, as shown in Figure 1, which was adapted from Marti 1990. The design implementation in SAP2000 assumes there are no diagonal cracks in the core. In such a case, a state of pure shear develops within the core, and hence the transverse shear force at a section has no effect on the in-plane forces in the sandwich covers. Thus, no transverse reinforcement needs to be provided, and the in-plane reinforcement is not enhanced to account for transverse shear.

The following items summarize the procedure for concrete shell design, as implemented in SAP2000:

1. As shown in Figure 1, the slab is conceived as comprising two outer layers centered on the mid-planes of the outer reinforcement layers.

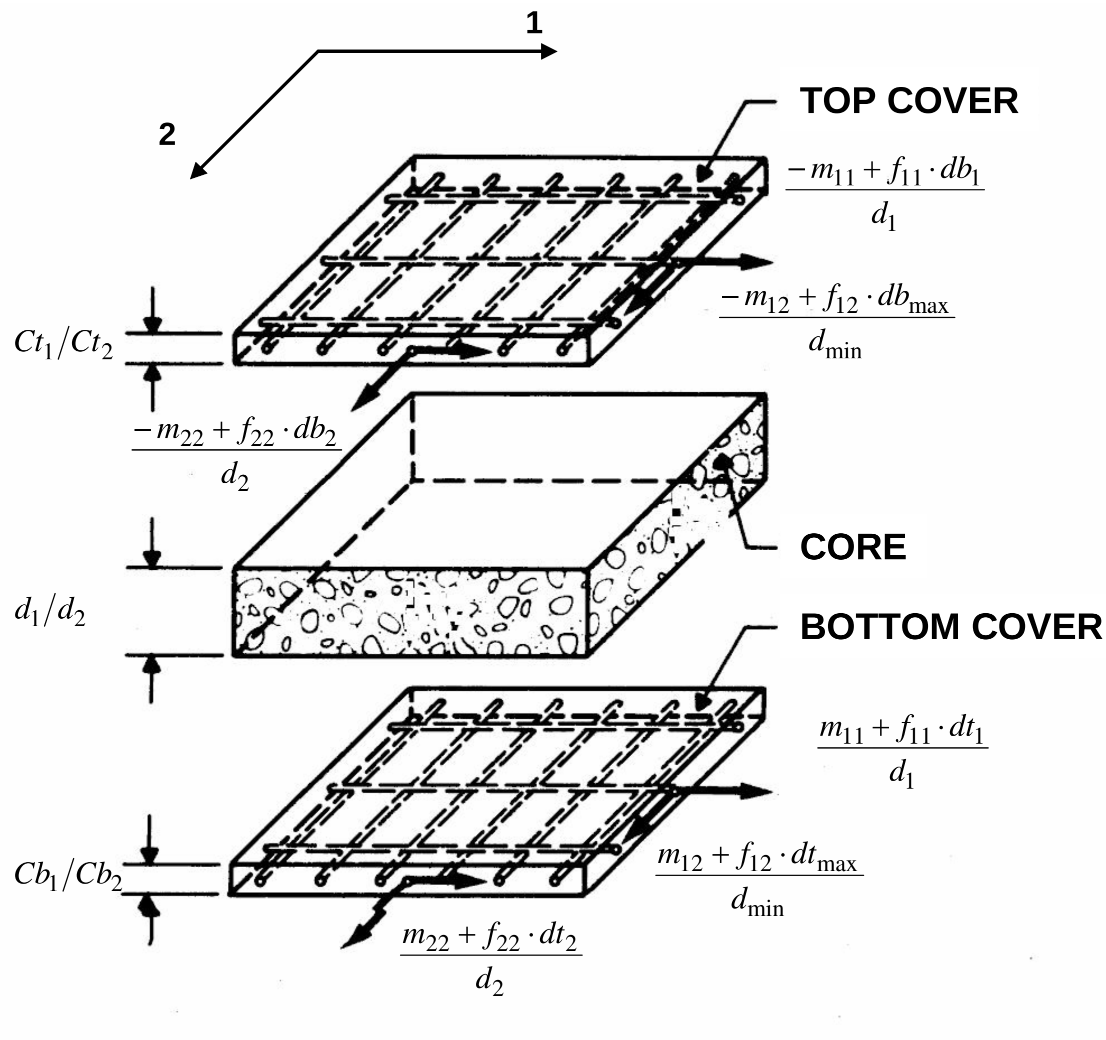

Figure 1: Statics of a Slab Element – Sandwich Model

2. The thickness of each layer is taken as equal to the lesser of the following:

   - Twice the cover measured to the center of the outer reinforcement.
   - Twice the distance from the center of the slab to the center of outer reinforcement.

3. The six resultants, f_{11}, f_{22}, f_{12}, m_{11}, m_{22}, and m_{12}, are resolved into pure membrane forces N_{11}, N_{22} and N_{12}, calculated as acting respectively within the central plane of the top and bottom reinforcement layers. In transforming the moments into forces, the lever arm is taken as the distance between the outer reinforcement layers.

4. For each layer, the reinforcement forces $NDes_1$, $NDes_2$, concrete principal compressive forces Fc_1, Fc_2, and concrete principal compressive stresses Sc_1 and Sc_2, are calculated in accordance with the rules set forth in Brondum-Nielsen 1974.

5. Reinforcement forces are converted to reinforcement areas per unit width Ast_1 and Ast_2 (i.e., reinforcement intensities) using appropriate steel stress and stress reduction factors.

Basic Equations for Transforming Stress Resultants into Equivalent Membrane Forces

For a given concrete shell element, the variables h, Ct_1, Ct_2, Cb_1, and Cb_2, are constant and are expected to be defined by the user in the area section properties. If those parameters are found to be zero, a default value equal to 10 percent of the thickness, h, of the concrete shell is used for each of the variables. The following computations apply:

$$dt_1 = \frac{h}{2} - Ct_1\ ; \quad dt_2 = \frac{h}{2} - Ct_2\ ; \quad db_1 = \frac{h}{2} - Cb_1\ ; \quad db_2 = \frac{h}{2} - Cb_2$$

$$d_1 = h - Ct_1 - Cb_1\ ; \qquad d_2 = h - Ct_2 - Cb_2\ ;$$

d_{min} = Minimum of d_1 and d_2

db_{max} = Maximum of db_1 and db_2

dt_{max} = Maximum of dt_1 and dt_2

The six stress resultants obtained from the analysis are transformed into equivalent membrane forces using the following transformation equations:

$$N_{11}(top) = \frac{-m_{11} + f_{11} \cdot db_1}{d_1}; \qquad N_{11}(bot) = \frac{m_{11} + f_{11} \cdot dt_1}{d_1}$$

$$N_{22}(top) = \frac{-m_{22} + f_{22} \cdot db_2}{d_2}; \qquad N_{22}(bot) = \frac{m_{22} + f_{22} \cdot dt_2}{d_2}$$

$$N_{12}(top) = \frac{-m_{12} + f_{12} \cdot db_{\max}}{d_{\min}}; \qquad N_{12}(bot) = \frac{m_{12} + f_{12} \cdot dt_{\max}}{d_{\min}}$$

Equations for Design Forces and Corresponding Reinforcement Intensities

For each layer, the design forces in the two directions are obtained from the equivalent membrane forces using the following equations according to rules set out in Brondum-Nielsen 1974.

$$NDes_1(top) = N_{11}(top) + Abs\{N_{12}(top)\}$$

$$NDes_1(bot) = N_{11}(bot) + Abs\{N_{12}(bot)\}$$

$$NDes_2(top) = N_{22}(top) + Abs\{N_{12}(top)\}$$

$$NDes_2(bot) = N_{22}(bot) + Abs\{N_{12}(bot)\}$$

Following restrictions apply if $NDes_1$ or $NDes_2$ is less than zero:

If $NDes_2(top) < 0$ then $$NDes_1(top) = N_{11}(top) + Abs\left\{\frac{[N_{12}(top)]^2}{N_{22}(top)}\right\}$$

If $NDes_1(top) < 0$ then $$NDes_2(top) = N_{22}(top) + Abs\left\{\frac{[N_{12}(top)]^2}{N_{11}(top)}\right\}$$

If $NDes_2(bot) < 0$ then $$NDes_1(bot) = N_{11}(bot) + Abs\left\{\frac{[N_{12}(bot)]^2}{N_{22}(bot)}\right\}$$

If $NDes_1(bot) < 0$ then $$NDes_2(bot) = N_{22}(bot) + Abs\left\{\frac{[N_{12}(bot)]^2}{N_{11}(bot)}\right\}$$

The design forces calculated using the preceding equations are converted into reinforcement intensities (i.e., rebar area per unit width) using appropriate steel stress from the concrete material property assigned to the shell element and the stress reduction factor, ϕ_s. The stress reduction factor is assumed to always be equal to 0.9. The following equations are used:

$$Ast_1(top) = \frac{NDes_1(top)}{0.9(f_y)}; \qquad Ast_1(bot) = \frac{NDes_1(bot)}{0.9(f_y)}$$

$$Ast_2(top) = \frac{NDes_2(top)}{0.9(f_y)}; \qquad Ast_2(bot) = \frac{NDes_2(bot)}{0.9(f_y)}$$

Principal Compressive Forces and Stresses in Shell Elements

The principal concrete compressive forces and stresses in the two orthogonal directions are computed using the following guidelines from Brondum-Nielsen 1974:

$$Fc_1(top) = N_{11}(top) + \frac{\{N_{12}(top)\}^2}{N_{11}(top)} \quad \text{if} \quad NDes_1(top) < 0$$

$$= -2 \cdot Abs\{N_{12}(top)\} \quad \text{if} \quad NDes_1(top) \geq 0$$

$$Fc_1(bot) = N_{11}(bot) + \frac{\{N_{12}(bot)\}^2}{N_{11}(bot)} \quad \text{if} \quad NDes_1(bot) < 0$$

$$= -2 \cdot Abs\{N_{12}(bot)\} \quad \text{if} \quad NDes_1(bot) \geq 0$$

$$Fc_2(top) = N_{22}(bot) + \frac{\{N_{12}(top)\}^2}{N_{22}(top)} \quad \text{if} \quad NDes_2(top) < 0$$

$$= -2 \cdot Abs\{N_{12}(top)\} \quad \text{if} \quad NDes_2(top) \geq 0$$

$$Fc_2(bot) = N_{22}(bot) + \frac{\{N_{12}(bot)\}^2}{N_{22}(bot)} \quad \text{if} \quad NDes_2(bot) < 0$$

$$= -2 \cdot Abs\{N_{12}(bot)\} \quad \text{if} \quad NDes_2(bot) \geq 0$$

The principal compressive stresses in the top and bottom layers in the two directions are computed as follows:

$$Sc_1(top) = \frac{Fc_1(top)}{2 \cdot Ct_1}; \qquad Sc_1(bot) = \frac{Fc_1(bot)}{2 \cdot Cb_1}$$

$$Sc_2(top) = \frac{Fc_2(top)}{2 \cdot Ct_2}; \qquad Sc_2(bot) = \frac{Fc_2(bot)}{2 \cdot Cb_2}$$

Notations

The algorithms used in the design of reinforcement for concrete shells are expressed using the following variables:

$Ast_1(bot)$	Reinforcement intensity required in the bottom layer in local direction 1
$Ast_1(top)$	Reinforcement intensity required in the top layer in local direction 1
$Ast_2(bot)$	Reinforcement intensity required in the bottom layer in local direction 2
$Ast_2(top)$	Reinforcement intensity required in the top layer in local direction 2
Cb_1	Distance from the bottom of section to the centroid of the bottom steel parallel to direction 1
Cb_2	Distance from the bottom of the section to the centroid of the bottom steel parallel to direction 2
Ct_1	Distance from the top of the section to the centroid of the top steel parallel to direction 1
Ct_2	Distance from the top of the section to the centroid of the top steel parallel to direction 2
d_1	Lever arm for forces in direction 1
d_2	Lever arm for forces in direction 2
db_1	Distance from the centroid of the bottom steel parallel to direction 1 to the middle surface of the section
db_2	Distance from the centroid of the bottom steel parallel to direction 2 to the middle surface of the section
db_{max}	Maximum of db_1 and db_2

d_{min}	Minimum of d_1 and d_2
dt_1	Distance from the centroid of the top steel parallel to direction 1 to the middle surface of the section
dt_2	Distance from the centroid of the top steel parallel to direction 2 to the middle surface of the section
dt_{max}	Maximum of dt_1 and dt_2
f_{11}	Membrane direct force in local direction 1
f_{12}	Membrane in-plane shear forces
f_{22}	Membrane direct force in local direction 2
$Fc_1(bot)$	Principal compressive force in the bottom layer in local direction 1
$Fc_1(top)$	Principal compressive force in the top layer in local direction 1
$Fc_2(bot)$	Principal compressive force in the bottom layer in local direction 2
$Fc_2(top)$	Principal compressive force in the top layer in local direction 2
f_y	Yield stress for the reinforcement
h	Thickness of the concrete shell element
m_{11}	Plate bending moment in local direction 1
m_{12}	Plate twisting moment
m_{22}	Plate bending moment in local direction 2
$N_{11}(bot)$	Equivalent membrane force in the bottom layer in local direction 1
$N_{11}(top)$	Equivalent membrane force in the top layer in local direction 1
$N_{12}(bot)$	Equivalent in-plane shear in the bottom layer

$N_{12}(top)$	Equivalent in-plane shear in the top layer
$N_{22}(bot)$	Equivalent membrane force in the bottom layer in local direction 2
$N_{221}(top)$	Equivalent membrane force in the top layer in local direction 2
$NDes_1(top)$	Design force in the top layer in local direction 1
$NDes_1(top)$	Design force in the top layer in local direction 2
$NDes_2(bot)$	Design force in the bottom layer in local direction 1
$NDes_2(bot)$	Design force in the bottom layer in local direction 2
$Sc_1(bot)$	Principal compressive stress in the bottom layer in local direction 1
$Sc_1(top)$	Principal compressive stress in the top layer in local direction 1
$Sc_2(bot)$	Principal compressive stress in the bottom layer in local direction 2
$Sc_2(top)$	Principal compressive stress in the op layer in local direction 2
ϕ_s	Stress reduction factor

References

Brondum-Nielsen, T. 1974. Optimum Design of Reinforced Concrete Shells and Slabs. Technical University of Denmark. Report NR.R.

Marti, P. 1990. Design of Concrete Slabs for Transverse Shear. *ACI Structural Journal.* March-April.