Le cercle et ses tangentes

a) calcul des tangentes : IT = IT′

$$= R \tan(\frac{\theta}{2}) = R \operatorname{cotan}(\frac{\alpha}{2}) \quad = OI \sin(\frac{\theta}{2}) = OI \cos(\frac{\alpha}{2})$$

$$= \frac{HI}{\sin(\frac{\theta}{2})} = \frac{HI}{\cos(\frac{\alpha}{2})} = \frac{TH}{\cos(\frac{\theta}{2})} = \frac{TH}{\sin(\frac{\alpha}{2})}$$

$$= \sqrt{HI^2 + TH^2} \quad = \sqrt{OI^2 - OT^2} \quad = \sqrt{IM \times IM'} = \sqrt{HI \times OI}$$

b) calcul du rayon : R

$$= IT \tan(\frac{\alpha}{2}) = IT \operatorname{cotan}(\frac{\theta}{2}) = OI \sin(\frac{\alpha}{2}) = OI \cos(\frac{\theta}{2})$$

$$= \frac{TH}{\sin(\frac{\theta}{2})} = \frac{TH}{\cos(\frac{\alpha}{2})} = \frac{OH}{\cos(\frac{\theta}{2})} = \frac{OH}{\sin(\frac{\alpha}{2})}$$

$$= \sqrt{TH^2 + OH^2} \quad = \sqrt{OI^2 - IT^2} \quad = \sqrt{OH \times OI} \quad = \frac{C^2}{8F} + \frac{F}{2}$$

c) calcul de la corde : TT′ = C

$$= 2R \cos(\frac{\alpha}{2}) = 2R \sin(\frac{\theta}{2}) \quad = 2\,IT \sin(\frac{\alpha}{2}) = 2\,IT \cos(\frac{\theta}{2})$$

$$= 2\,OH \operatorname{cotan}(\frac{\alpha}{2}) = 2\,OH \tan(\frac{\theta}{2}) \quad = 2\,HI \tan(\frac{\alpha}{2}) = 2\,HI \operatorname{cotan}(\frac{\theta}{2})$$

$$= 2\sqrt{IT^2 - HI^2} \quad = 2\sqrt{OT^2 - OH^2} \quad = 2\sqrt{OH \times HI} \quad = 2\sqrt{2 \times R \times F - F^2}$$

e) calcul de la flèche : OM-OH = HM = F

$$= R\,(1 - \cos(\frac{\theta}{2})) = R\,(1 - \sin(\frac{\alpha}{2})) = R - \sqrt{R^2 - C^2/4}$$

f) longueur de l'arc $\overset{\frown}{TMT'} = \dfrac{\pi\, R\, \theta}{180°}$

g) aire du secteur circulaire OTMT' = $\dfrac{\pi\, R^2 \theta}{360°}$

h) aire du segment circulaire TMT' = $\dfrac{\pi\, R^2 \theta}{360°} - \dfrac{R^2 \sin(\theta)}{2}$

i) aire de la pointe ITMT' = IT×R - $\dfrac{\pi\, R^2 \theta}{360°}$

j) angle $\theta = 2 \arcsin\left(\dfrac{C}{2R}\right)$

Dans cet exemple (unité mm) :

$\alpha = 60°$ $\theta = 120°$

R	50
IT	86.603
OI	100
HI	75
TH	43.301
OT	50
IM	50
IM'	150
OH	25
C	86.603
F	25
lg arc TMT'	104.720
aire OTMT'	2617.994
aire TMT'	1535.462
aire ITMT'	1712.133

